

Characteristic Of The Incomplete Mathematical Connections In Understanding Of Algebraic Problem

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Abstract: This study aims to describe the characteristics of mathematical connections when undergraduate students are understanding the algebraic problem. The research question is "What characteristics of mathematical connections when undergraduate students understanding algebraic problems?". This research is a qualitative research with explorative descriptive method. In the selection of subject, this study involved 20 first-degree students of mathematics education courses. To describe the characteristics of the incomplete mathematical connections, it is enough to analyze data for 4 subjects. The main instrument of the research is the researcher himself, while the supporting instrument is a problem solving task sheet and interview protocol. The result of research showed that incomplete connections appeared in undergraduate students when understanding algebraic problems. It can be concluded that the characteristics of students' incomplete connections when understanding the algebraic problem can be distinguished become two types i.e. the simple incomplete connections and the complex incomplete connections. The simple incomplete connection occurs when incompleteness of connection only appears on a connection block and does not continue on the next block of connection. While the complex incomplete connections occurs when incompleteness of connection appears on some blocks of connections.

Index Terms: characteristic, mathematical connections, incomplete connections, understanding, algebraic problem.

1 INTRODUCTION

Mathematical connections can be illustrated as structured networks such as spider webs (Hiebert & Carpenter, 1992). The intersection or node of the network can be thought of as a representation of information or knowledge, while the thread or path that links it can be viewed as a link or a connector. Thus there are two main parts in the mathematical connection, namely the connected components and connectors or connecting lines. The first major part of the mathematical connection, the connected component, is a representation of one's own knowledge. Mathematical knowledge is generally divided into conceptual knowledge and procedural knowledge. Conceptual knowledge is essentially about understanding relationships between ideas and mathematical concepts (Hiebert & Lefevre, 1986). Procedural knowledge in its purest form focuses on the symbolization, skills, rules and step-by-step algorithms used in completing a mathematical task. One should learn concepts at once with procedures so they can make connections. The second main part of the mathematical connection is the connector or connection path. Mathematical connectors include ideas, such as functions, matrices, algorithms (procedures), graphs, variables, comparisons, and transformations (Coxford, 1995). So connectors are emerging ideas in relation to studying the wide spectrum of topics. By using connectors emerging ideas can be developed into new concepts/ideas as a result of connections. To be able to examine more deeply the nature of the connections, the mathematical knowledge that is connected needs to be reviewed in more detail. In this case, mathematical knowledge is also viewed from the direct object of mathematical aspects. According to Gagne, the direct object of mathematics includes facts, concepts, principles, and procedures or skills (Bell, 1978).

Mathematical facts are an agreement in mathematics, such as mathematical symbols. Mathematical skills are operations and procedures that are expected to provide speed and accuracy. Concepts in mathematics are abstract ideas that are able to classify each object or event as an example or not an example of the abstract idea. Principle is a set of concepts along with the relationship between the concepts. Problem solving is one of the indirect objects of mathematics, alongside other indirect objects, namely the ability to think logically, positive attitudes toward mathematics, perseverance, and thoroughness (Bell, 1978). Problems are questions or questions that students can understand and have an interest in but can't be answered instantly with the routine procedures they already know. So a question is a problem if the question is challenging to answer and the answer can't be done routinely (Sutarto, et al., 2019). Solving problems is the process of accepting the challenge of answering the question that is the problem (Hudojo, 1988; Hastuti, et al, 2020). A problem for college students if given to an elementary school student does not include a problem because the problem will not be understood and will not challenge the answer. The well-known framework in the problem solving is the Polya's framework. According Polya (1988) there are four phases in problem solving, namely: (1) to understand the problem, (2) to make a plan, (3) carry out the plan, and (4) look back. These four phases are a framework in problem solving that is generally implemented and are relatively sequential. Mathematical connections serve as a tool in problem solving. According to Hodgson (1995), "connection can't solve problems directly, but they enable problem solver to do so". In other words, the connection facilitates students in troubleshooting, as a tool used to solve problems. When students solve problems, the information on the problem must be connected to the knowledge that has been mastered. Furthermore, the students explore or recall their knowledge related to the information and questions of the problem. In the Polya's framework, this connecting activity is done at each phase of problem solving. Mathematical connections can also be described as components of a connected scheme or group of schemes in the mental network. Marshall (1995) suggests

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that the characteristic of the scheme is the existence of a connection. The more connections, the greater the compactness and strength of the scheme. Coxford (1995) states that mathematical connections are assumed to have three related aspects: (1) mixing themes, (2) mathematical process, and (3) mathematical connectors. Research on mathematical connections has been done before at all levels of education, ranging from Elementary School (Chung, 2004; Fox, 2006; Makar, 2007; Palomar et al., 2006; Wilburne & Napoli, 2008), Junior High School (Businskas, 2005 Gerson & Walter, 2007; Lopez, 2001), Senior High School (Bosse, 2003; Ketterlein-Gein et al., 2008; Monaghan & Otmantar, 2006; Ozgen, 2013; Stemhagen, 2008; Uptegrove & Maher, 2005), until at university (Adlakha & Kowalski, 2007; Bilotski & Subbotin, 2009; Kondratieva & Radu, 2009; Presmeg, 2006; Yantz, 2013). However, the research has mostly revealed the existence of mathematical connections and categorize them, while research on the characteristics of thinking that focus on the characteristics of mathematical connections is still not much revealed in the studies. Previous research by the researcher resulted two types of connections errors, namely conceptual connections errors and procedural connections errors (Arjudin et al, 2016). The appearance of errors on the mathematical connection indicates the incompleteness of the connection. This study explores the characteristics incomplete connections at the stage of understanding problems when students do algebra problem solving. This characteristic is seen from components that are connected whether appropriate or not, how to or whether they connect whether appropriate or not, and the results of whether the concession obtained whether appropriate or not. The stage of understanding the problem is a very important step in solving the problem, because it underlies the next steps of solving the problem.

2 METHOD

This research uses qualitative approach with explorative descriptive method. Qualitative approaches are considered appropriate because of their compatibility with qualitative research characteristics, such as research carried out in the field in a natural setting. Researchers face to face with respondents/subjects in conducting data collection research through documentation, behavioral observation, and interviews. This indicates that the researcher serves as a key instrument (researcher as key instrument). This research is included in the theme of thought process research. This study explores the characteristics of students' mathematical connections when solving algebraic problems. The characteristics of the mathematical connections are expressed by describing the characteristics of mathematical connections. In the selection of research subjects, this study involved 20 first year students of Mathematics Education Study Program at FKIP Universitas Mataram. To determine the subject of research, the researcher gives the problem solving task sheet to the prospective research subjects. The subject candidate is working on a problem-solving task with think aloud. As long as the subject candidate completes the problem-solving task, audio and visual recording are performed, while the researcher also observes and makes important notes in the field. Based on the results of problem solving conducted analysis and interviews, to determine the students who qualify as research subjects. Students who qualify as research subjects are students who can make mathematical

connections. This capability is shown by recording think aloud, written answers from problem solving sheets, interview results, and field notes.

2.1 Instruments

The main instrument of the research is the researcher himself, while the supporting instrument is a problem solving task sheet, interview protocol, and equipped with tape recorder. This instrument aims to determine the ability of undergraduate students in solving problems in which using the ability of mathematical connections. Prior to use, this problem solving task sheet was first validated by two expert validators. The problem given to the research subject is an algebraic problem containing the tasks or commands for planning the construction of the house and yard from the vacant lot and the garden and calculating the required cost and sketching the drawing. The question or command is preceded by a description of what is known as a prerequisite to the question on the matter accompanied by a table related to the component and amount of financing

2.2 Data Collection and Analysis

Data analysis procedures are more operational, covering the following stages: (1) conducting interview transcripts, field notes, sorting and compiling the data into different types depending on the source of information, (2) doing data reduction by focusing on data on mathematical connection characteristics and data that have the value of the development of the problem under study, (3) coding is the process of segmenting and labeling text to form broad data descriptions and themes in the data, (4) illustrating structure of student connections when solving algebraic problems, (5) analysis of connection characteristics, (6) conclusions.

3 FINDINGS

The result of research showed that incomplete connections appeared in students when understanding algebraic problems. The incompleteness of a mathematical connection is indicated by the inappropriateness of the connected component or the inappropriateness of the connection path or the inappropriateness of the result of the connection. This incomplete connection may occur on a single connection block or occurs on more than one connection block. Thus, it can be concluded that the characteristics of students' incomplete connections when understanding the algebraic problem can be distinguished two type i.e. the simple incomplete connection and the complex incomplete connection

3.1 The simple incomplete connections

The incomplete connections at the step of understanding the problem for Subject 1, which is called S1 only, illustrated by the part of his thinking structure in Figure 1 below.

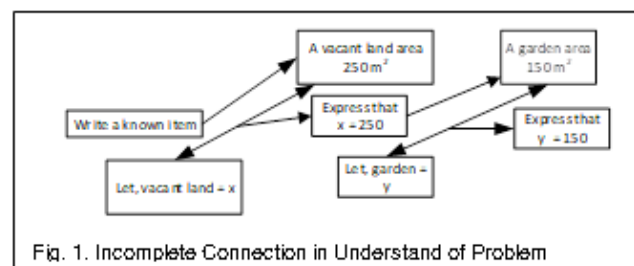


Fig. 1. Incomplete Connection in Understand of Problem

S1 connects existing problem information with improper idea, in this case existing information about empty land area of 250 m² connected with variable x which states the empty land so directly generate the value of variable $x = 250$. Likewise existing information about the area of the garden 150 m² is connected to variable y which states the area of the garden so that it directly produces the variable value $y = 150$. The incompatibility of this connection does not impact sustainably because at the planning stage of completion, S1 raises another variable whose connection is in accordance with the amount to be searched for value. The incomplete connections at the step of understanding the problem for Subject 2, which is called S2 only, illustrated by the part of his thinking structure in Figure 2 below

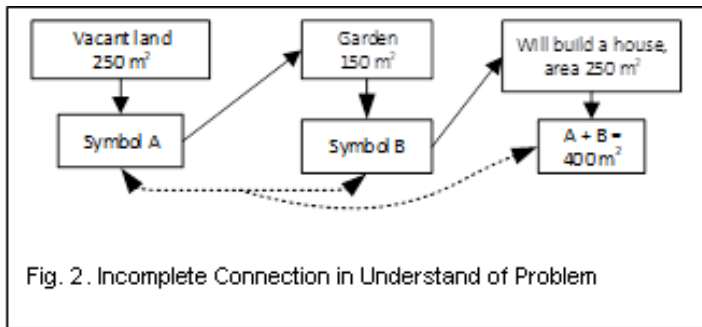


Fig. 2. Incomplete Connection in Understand of Problem

Mathematical connections made by S2 is to connect the existing problem information with the concept/idea is not right, in this case existing information about the empty land area of 250 m² connected with the symbol A. Similarly information about the 150 m² of garden area connected with the symbol B. The emergence of such connections because S3 until the stage has not been able to understand the problem, especially about the components in question. This incomplete connection does not have an on going impact, because at the planning stage the S3 settlement is not fixated on the variable annotation strategy but also uses a trial strategy. Based on the two mathematical connections performed by S1 and S2 respectively, the following characteristics of mathematical connection are obtained. At the stage of understanding the problem, a known problem information value is connected to a symbol/ variable. The result of this connection is that the value of the variable is less meaningful, so less support for the next step in the planning phase of the settlement. In this mathematical connection the components are not connected incorrectly, but the connection path is not appropriate, that is connecting the quantity of the known value with the variable, so that the resulting connection is also not appropriate. Such mathematical connections are called incomplete connections. The incompleteness of the mathematical connection only occurs in a single connection block and does not have an ongoing impact, as soon the error is realized and the subject fixes it immediately. Thus the mathematical connection is called a simple incomplete connection. The simple incomplete connection scheme can be illustrated in Figure 3.3 below.

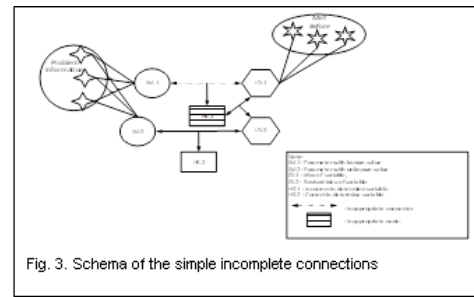


Fig. 3. Schema of the simple incomplete connections

3.2 The complex incomplete connections

The incomplete connections at the step of understanding the problem for Subject 3, which is called S3 only, illustrated by the part of his thinking structure in Figure 4 below.

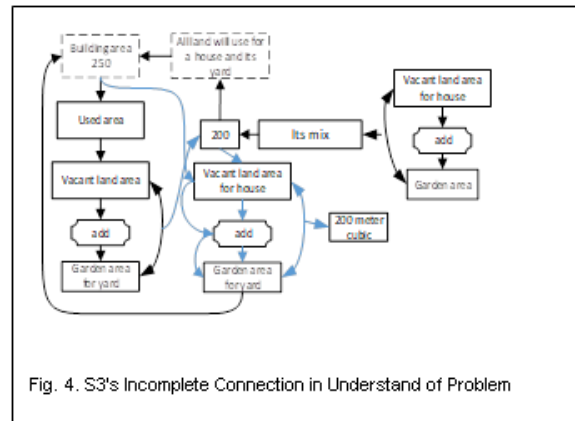


Fig. 4. S3's Incomplete Connection in Understand of Problem

Mathematical connection done above is in order to understand the problem information in the form of vacant land used for home building connected to the area of garden used for the yard. Incomplete connections are indicated by incompleteness, which implies inaccuracy, in revealing the components that are connected. First, the connected area of the garden is not the area of the garden used for the yard. The second, which is connected to a vacant lot of land, not the area of the garden used for the yard. In the third repetition, the corresponding connections are the area of vacant land used for house building and the vacant land used for the yard. Incomplete connections are said to be not simple because of these connection errors occur repeatedly. The incomplete connections at the step of understanding the problem for Subject 4, in future which is called S4 only, illustrated by the part of his thinking structure in Figure.5 below.

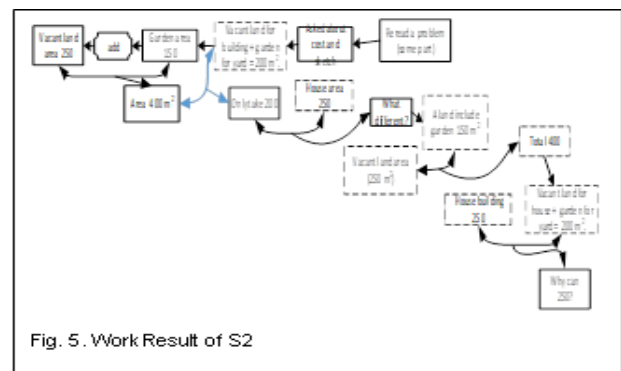


Fig. 5. Work Result of S2

In understanding the problem, S4 repeats reading some parts

problem into the appropriate scheme which have been formed through the process of accommodation earlier. This is indicated by the subject who is aware of his mistakes and improves on the next steps. Considering from the understanding, in Polya's opinion (Meel, 2003), the subject concerned is at the level of mechanical understanding (mechanical) because it applies only the method he remembers. If viewed from the understanding by Lehman (1977), it reflects the lack of knowledge about the application, meaning, and logic in the understanding of mathematics. This is evident from the lack of applicable methods, and their application is mechanical and less meaningful, and the logical relationship is also less so that errors occur repetitively and are not immediately realized, thereby further impacting on the subsequent steps of problem solving. Considering from the theory of assimilation and accommodation, the incomplete connection is not simpler than this process of accommodation more than the assimilation process. At first the subject did not have a schema / knowledge structure that matches the problem information received. This is demonstrated by the lack of understanding of the problem followed by repeated reading. Next comes the process of accommodation where the subject forms a new structure to adjust to the stimulus received. This is indicated by the subject of associating one problem with another information from the problem. After that there is a process of assimilation, which is the integration of received stimulus into the appropriate scheme that has been formed through the process of accommodation earlier. This is indicated by the subject being able to correct the error, although the mistakes made can be said to be complex because it impacts on the next steps..

5 CONCLUSION

The conclusion of this research that mathematical connections occur when the subject raises the components of the connection and connects it through a connection path or so-called connector, resulting in an idea/concept. Part of a mathematical connection consisting of connection components, connectors, and connection results is called a connection block. Incomplete connection occurs when the component is in the form of improper concepts/ideas or the connection path may be incorrect/incorrect or an inappropriate connection result may occur. Incomplete connection characteristics can be divided into two types, i.e, a simple incomplete connections and a complex incomplete connections are. The simple incomplete connection occurs when the characteristics of the incomplete connection appear on a single connection block and do not continue into the next connection block. Whereas if the incomplete mathematical connection occurs in a connection block and continues on the next connection block, then called the complex incomplete connections.

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