PAPER • OPEN ACCESS

Shifting of perfective metacognitive activities in solve math problems

View the article online for updates and enhancements.

Shifting of perfective metacognitive activities in solve math problems

I D Hastuti[1], Surahmat[2], Sutarto[3] and Dafik[4]

[1]Primary Teacher Education, FKIP Muhammadiyah University of Mataram, Mataram, Indonesia
[2] Mathematics Education, Malang Islamic University, Malang, Indonesia
[3] Mathematics Education, FPMIPA IKIP Mataram, Mataram, Indonesia
[4] Mathematics Education, University Of Jember, Jember, Indonesia

Corresponding author e-mail[1]: intandwihastuti88@gmail.com

Abstract. This study aims to describe the shift in the perfective metacognitive activities of students in solving mathematical problems. The subjects in this study were students who experienced a shift in the perfective metacognitive activities of students in solving mathematical problems. This research is a qualitative research with a descriptive exploratory approach. The instrument used was a mathematical problem solving task type MEA (Model Eliciting Activities), interview guidelines, and questionnaires. Data collection procedures in this study consisted of five stages. In the first stage, the researcher asks each student to solve the problem given while thinking alouds. The second stage, students are asked to solve the same problem as a group while discussing with two other students, then researchers observe and listen to the results of think alouds as well as the results of conversations during group discussions from a computer screen. The third step is giving the questionnaire. The fourth stage is task-based interviews to explore information that needs to be confirmed from the results of think alouds and questionnaires. Next to the fifth stage, researchers analyzed data from student work outcomes, think alouds, questionnaires, recorded conversations of students during discussions, and interviews. Based on the results of the study, there were students who experienced a shift in perfective metacognitive activity in solving mathematical problems.

1. Introduction

Metacognition and problem solving are important aspects that students must possess. Metacognition arises when individuals encounter unknown problems, uncertainties, or questions [9]. Metacognition is an important dimension of problem solving because metacognition includes awareness of one's thinking related to problems, monitoring and regulation of cognitive processes, and the application of heuristics [1]. Metacognition plays an important role in problem solving because metacognition can help problem solver to recognize problems that need to be solved, see what the problem really is, and understand how to achieve the goal or solution [10].

According to Wilson and Clarke [13], metacognition leads to the awareness of one's thinking, the evaluation of one's thinking, and the setting of one's thinking. It further explained that the definition is consistent with the existing literature, and at the same time extends from the definitions described by previous experts. Furthermore, Magiera and Zawojewski [11] suggest that there are three types of metacognitive activity: metacognitive consciousness, metacognitive regulation, and metacognitive

evaluation. The research of Magiera and Zawojewski refers to the framework of Wilson and Clarke [13].

Based on the opinion of the experts above, Hastuti [5] concluded that metacognition and metacognitive activity have the same meaning that is thinking of what has been thought. The term metacognitive activity has a broader meaning that includes metacognitive awareness, metacognitive regulation, and metacognitive evaluation [5]. Since the term metacognitive activity has a broader meaning, this study uses the term metacognitive activity, but in reference it still uses the terms used by previous experts on metacognition and metacognitive activity.

This research is a qualitative research with grounded theory type. The research was conducted on 21 October 2017 in Schools. Research is done by giving the problem of math type MEA (Model Eliciting Activities) that is open problem related to decision making to choose which hotel is best. This MEA type problem is given to students. Before we present it in more detail, let us look at the conceptual underpinnings of the research.

2. Literature Review

Previous researchers have reviewed and studied metacognition [7] [10] [11] [12]. The results of the study by Kapa [11] suggest that learning environments that provide metacognitive support during the problem-solving process at each stage are significantly more effective than learning environments that provide metacognitive support only at the end of the process. Magiera and Zawojewski [12] identify and characterize social-based and self-based contexts related to metacognitive activity that are coded as metacognitive awareness, metacognitive regulation, and metacognitive evaluation. This study yields three characteristics of a social-based context that is, 1) interpreting various perspectives significantly, 2) engaging in meaningful explanations, and 3) seeking mathematical agreements. Further characteristics of the self-based context are 1) seeking personal satisfaction, 2) making quantitative experience-based judgments, and 3) using personal projections. Furthermore, Mokos and Kafoussi [11] examine the spontaneity of metacognitive monitoring and control functions of fifth grade students in completing three types of mathematical problems, which are open-ended, authentic, and complex problems. The results of this study indicate that the spontaneous metacognitive strategies that appear in each type of problem are traced through students' verbal reports. Furthermore, Kuzle [10] describes the problem-solving behavior of two prospective teachers in solving non-routine geometry problems individually.

One's metacognitive activity can evolve through social interaction, where conversation can serve as a tool that supports the emergence of metacognitive activity ([12]. Social metacognition requires mutual relationships and the involvement of other members in a group to solve common problems. Social metacognition arises when one group member contributes to discussing how to cultivate a task and influence other members of the group so that the other members of the group respond and develop it [6]. Thus social metacognition arises when one group member puts forward the problem-solving process and the other members of a group respond to, respond to, and develop ideas from their discussion friends.

Some other experts have also conducted research related to metacognitive activity in group or social discussions. Research conducted by Chiu and Kuo [2] reveals that social metacognition has many benefits including 1) can distribute metacognitive needs, 2) make metacognition more visible, 3) increase individual cognition, 4) encourage mutual scaffolding , 5) Encourage greater motivation. Goos, Galbraith, and Renshaw [4] examined the pattern of social interaction of middle-class students mediated by metacognitive activity. Magiera and Zawojewski [11] identify and characterize social-based and self-based contexts related to metacognitive activity that are coded as metacognitive awareness, metacognitive regulation, and metacognitive evaluation.

Previous studies [4] [11] [6] still have not revealed the shift in metacognitive activity of students' perfective from individual to social in solving math problems. The shift in perfective metacognitive activity occurs when students get the effect of group discussion so that students re-examine their mathematical thinking and revise their initial solution in resolving the problem [6]. The ideas of a

discussion companion leads one to rethink what he or she has thought therefore, she or he perform metacognitive awareness, metacognitive evaluation, and even metacognitive regulation. Because previous studies have not revealed the shifting of students' perfective metacognitive activity, this study aimed at describing the shift in metacognitive activity of students in solving mathematical problems.

3. Methodology

3.1 Subject
The process of selecting research subjects incomplete vern in 45 students of class XI with details of 24 students of SMAN 1 Malang and 21 students of SMAN 3 Malang. From that process, we obtained 11 subjects who experienced the shift of metacognitive activity perfective.

3.2 Data Instruments
The data Instruments of this research are mathematical problem solving task of MEA (Model Eliciting Activities), interview guides, and questionnaires. MEA is a type of open problem that requires the development of a mathematical model and requires enough challenges so that group members must be involved to decide, test, and revise their initial solution which in turn leads to monitoring, evaluating the effectiveness of their initial solution, and making decisions. MEA in this study is open problem related to decision making to choose which hotel is best.

3.3 Design And Procedure
The first stage is the researcher asks each student to solve a given problem while thinking alouds (voicing what is thought). In the second stage, students are asked to solve the same problem as a group while discussing with two other students. When students work individually and discuss, the researcher observes and listens to the results of think alouds and the results of conversations during group discussions from a computer screen. The third stage is the provision of questionnaires aimed at indicating whether or not there are metacognitive activities of students that arise when solving problems. The fourth stage is task-based interviews to explore information that has not been obtained or information that needs to be confirmed based on the results of think alouds and questionnaires. Next to the fifth stage, researchers analyzed data from student work outcomes, think alouds, questionnaires, student conversation results during group discussions, and interviews.

4. Results And Discussion
Data from subjects with perfective metacognitive activity shifts were analyzed based on student work outcomes, think-alouds outcomes, questionnaires, field notes, conversation results during group discussions, and interviews. Subjects that fall into the category of perfective metacognitive activity shift are S1 and S2.

4.1 Expose Data and Subject S1 Thinking Structure
At the stage of understanding the problem, subject S1 performed metacognitive awareness activities, visible from the thought S1 is a question related to the total registration fee and the hotel that became the best choice for the basketball team. Furthermore S1 re-think it by reading back the problem and consider the implementation of the game on June 15-16, thus S1 realized that the total cost of the tournament is the registration fee and hotel fee for 2 days stay a 1 night. This fact is evidenced from the results of think aloud S1 and excerpts of interviews between researchers with S1.

S1: (begin to read a problem). No. 1. The Hotel is millenium one, which price is average 1090.000 per room of a person, room capasity maximum is 4 persons, it is 10 km distance, with 4 restaurant number, and the hotel facilities only swimming pool. Pradana hotel, average price per night per room 630,000, maximum person per room 2 persons, 15 km distance, many restaurants 7, hotel facilities swimming pool and playground. Santika hotel, average price per person per room 1,320,000, maximum person per room 4 persons, distance 5 km, restaurant lot 2, hotel facilities

only playground. Shortly ... briefly ... (pause, then read The total cost of the tournament, meaning the first registration fee 1,500,000, is for 2 days 1 night due to 15-16 June, so the Hotel Millennium is 1,090,000 or Hotel Santika 1,320,000? because these two hotels are free of charge transportation (metacognitive awareness).

Furthermore the results of think alouds is reinforced by the results of interviews with subject S1 as follows. I: Interviewer, S1: S1 subject

I : What was your first thought, sister' after reading this problem?

S1: So this is why searching the best hotel for the team and make the stay day 1 night. From the price if indeed seen Pradana Hotel is cheap but with a distance of 15 km from where the game was far away and need transport costs. Whereas if at the Millennium hotel it is only 10 km, but the cost of transportation is free. if in Santika is near but the price is very expensive although free of charge transport, but yes, the term kayak worth so when compared to the Millennium is more worth in Millennium (metacognitive awareness).

I : Okay. Why did you read the problem over and over again?

S1: That ... to find somethings ... the point is actually where... (metacognitive awareness)

Furthermore, at the planning stage, the subject S1 performs metacognitive regulatory activity, which is indicated by the statement. If for example like this, it is from the possibility that there can find the cheaper and more comfortable if we let us again on the road, and that fall more worthed. Based on this statement, the thought of S1 is to make plans to find a cheaper and effective hotel. Furthermore S1 re-think it by choosing a strategy that is making three choices of stay, including if staying a team in a separate hotel and then compare the cost. This fact is evidenced from the results of interviews between researchers and subjects S1, and the following results of interview.

I : Why do you choose the way she wrote on this answer sheet?

S1: If for example like this, it's from the possibilities that there can be a cheaper and better looking if we say we are on the way, and that fall is more worthed (metacognitive regulation). So he pays cheap, and reciprocity to us is also profitable. No need far, the hotel is pretty good facilities and the cost of transportation is free.

At the stage of executing the plan, the S1 subject carries out a plan that is made by making three options to stay and compare the cost. First choice if basketball team rent 3 rooms in Millenium, second choice if basketball team rent 2 rooms in Millenium and 1 room in Pradana, and third choice is if basketball team rent 5 rooms at Pradana Hotel. then further S1 compares the lodging costs of the three options that have been made. At this stage, S1 also performs metacognitive evaluation activities. visible from the thought S1 is related to lease 3 rooms by the team at the Millennium Hotel and if the team rented in two separate hotels ie rent 2 rooms at Hotel Millenium and 1 room at Hotel Pradana. Furthermore, the S1 rethinks it by considering that if the team stays at two separate hotels at the Millennium Hotel and Pradana then it costs a little more than a team staying at the Millennium Hotel and added if the team stays in two separate hotels it is necessary to consider transportation costs again. Here are the results of think alouds and interviews between researchers with S1.

S1: Okay decision, if for example 3 rooms in Millennium, that would be 1.990.000 times 3janya 3270.000 minimal transportation costs. Continue if for example 2 rooms in Millenium and 1 room in Pradana that means 2,180,000 plus 630.000 equals 2,820,000 plus plus. The difference is 3270.000 have not eaten, yet transport. however, if the transport is like this, it is not worthed. It also 2 peoples were separated. It was crazy when it was separated (metacognitive evaluation).

Further the results of think alouds S1ini also reinforced by the interview as follows.

I : Okey just keep on how sister how to solve this problem?

S1: So the first one is caught in expensive expensive means no need to be selected. Finally there are 3 options, the first option to use 3 rooms in Millenium, this means enough for 12 people, or maybe the option to 2 rooms is enough for 10 people. so that there are 2 rooms at the pick of the Millenium hotel and 1 more room at Pradana Hotel. however, if separated cause obstacles in the cost of transport, there is a story must pay, there is no need to pay the transport so mending not necessarily lah. then, the third option is in Pradana with the price asusmsi cheaper so that, ultimately the price, for example calculated for 5 rooms because this can only make 2 people, it price is not much different than the one in Millennium that only need 3 rooms. Not to mention there is no transportation cost (metacognitive evaluation).
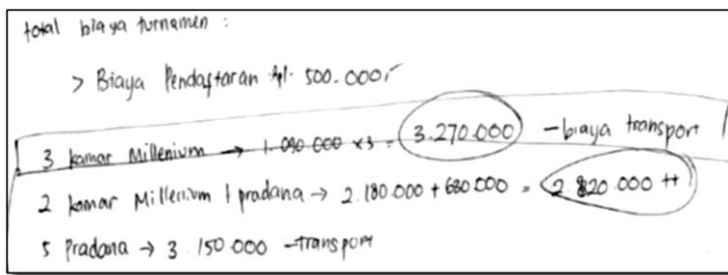
Total tournament cost
Registration fee

2 millenium rooms&1 pradana room
2.180.000+680.000=2.820.000++
5 pradana rooms →3.150.000 -
transport

Figure 1. The Result Of Work From S1 When Solving The Problem

The Think Subject S1 structure when solving problems individually can be illustrated in Figure 2.
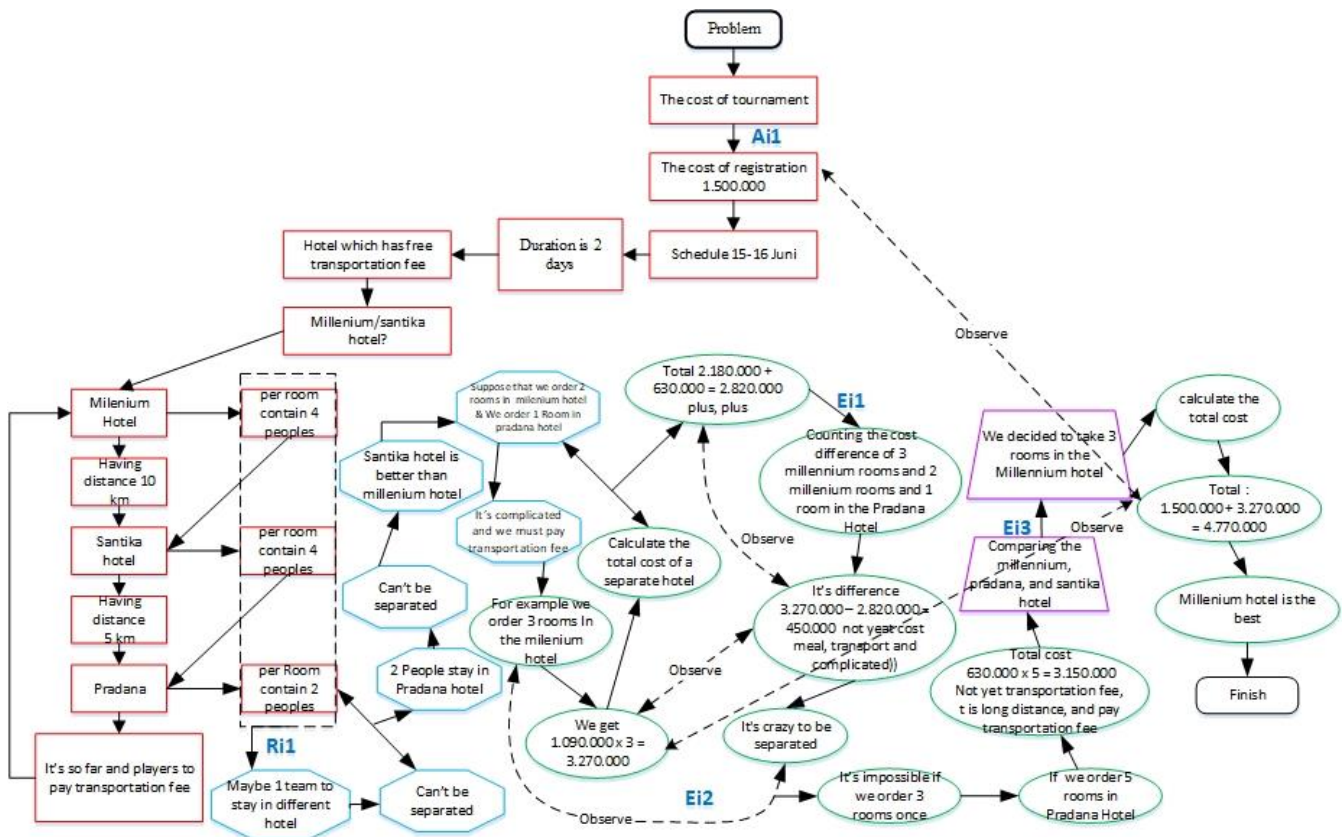

Figure 2. Chart of S1 Thinking Structure When Solving Problems Individually
**Table 1.** Caption Code Componen of Thinking Structure of S1 When Solving Problems Individually

Code	Descriptions	Code	Descriptions

[red rectangle]	Phase of understand the problem when solving the problem individually	[oval]	Components of lost thinking structures
[blue octagon]	Phase of devise a plan when solving problems individually	[arrow]	The order of activity when solving the problem
[green ellipse]	Phase of carryout the plan when solving problems individually	Ai	Individual metacognitive awareness
[trapezoid]	Phase of look back when solving problems individually	Ri	Individual metacognitive regulation
[orange rectangle]	Phase of understand the problem when gaining influence from group discussions	Ei	Individual metacognitive evaluation
[blue filled octagon]	Phase of devise a plan when gaining influence from group discussions	As	Social metacognitive awareness
[green filled ellipse]	Phase of carry out the plan when gaining influence from group discussions	Es	Social metacognitive evaluation

4.2 The Exposure Data of Subject S1 When Problems Solving in Groups

When one of the friends from S1 with the initials C said about the cost of each player, the subject S1 performs metacognitive awareness activities marked by the statement "oh yes yes ... this is because the cost per player required". Based on this statement, the thought of S1 is the first of the S1 discussion friends with the initials C associated if a basketball team consists of 9 players and 1 coach, then the coach participates to pay the cost of accommodation or not. Next S1 thinks back to it so S1 realizes that he has not calculated the cost per player. Here is the transcript of the discussion between the subject of S1 with 2 friends discussion with initials C and Y.

C : does every coach team participate, right?
S1: oh yes yes ... this is because the cost of each player (metacognitive awareness)
Y : yes, the first count of each team is then divided by 10
C : but the coach does not pay, calculate the cost of each player
S1: whether each player yes, means player doang
C : So, is it split 9?
S1: coach come to pay, so the cost is cheaper
C : So. it is divided by 10?

Conversation discussion subject of S1 with two of his friends confirmed by interviews conducted by researchers with subject S1. At the time the disco-friend of the subject S1 with initial C discusses whether the trainer is charged to pay or not, the subject S1 performs metacognitive awareness activities marked by the statement "Yeah it's just another fit to say this should be divided by 9 players only or coach in free wrote or directly divided by 10 players. Based on this statement, the thought of S1 is related to the question of the friend of the discussion that to determine the cost of each player, the coach is charged to pay or not. Next S1 think back to what was thought related to the question of his discussion friends so that S1 realized that he has not calculated the cost per player. Subject S1 also performs metacognitive evaluation activities as evidenced by the final statement I still say if suppose directly divided by 10 people, that result is more fair. If one paid one ndak then the other is burdened. Finally I decided directly for 10 people only. "Based on this statement, the thought of S1 is related to determining the cost per player. S1 then rethink what has been thought by giving the reason that to be more fair then the coach also pay the cost of accommodation so to determine the cost per player is the total cost of registration and hotel divided by 10. Here are the results of interviews between researchers and the subject S1.

P : Okey then, after the discussion, what are you thinking about this problem again?

S1: Yes it's fit again the other talking, is divided 9 players doang or coachnya paid or directly divided by 10 players? (metacognitive awareness)

Q : Then finally how you are

S1: Finally I still say that if for example directly divided by 10 people that fall more fair. If one paid one ndak then the other is burdened. Finally I decided directly for 10 people only. (Metacognitive evaluation)

Q : Did you think you did not do it when you were alone, this is divided into 10 people or 9 people?

S1 : The forgetfulness was bu, do not think about the cost per player

   During the discussion, subject S1 reconsidered the cost of eating so that the subject S1 performs meta-cognitive awareness activities and metacognitive evaluation. Metacognitive awareness performed by S1 is marked by the statement "this is still not the cost of eating". Based on this statement, the thought of S1 is related to total expenses of Rp 4.770.000,00. Furthermore, S1 re-think what has been thought related to the total cost of expenditure, which is to realize again that the cost of expenditure of Rp 4.770.000,00 still not including the cost of eating. The subject of S1 also performs a metacognitive evaluation marked by the statement "according to my experience, this lunch is the same as dinner". Based on this statement, the thought of S1 is related to a statement from a friend of S1 discussion with initials C that the cost of eating has been borne by the hotel. S1 rethink it by assessing the statement from a friend of the discussion that based on his experience, the hotel only bear the cost of breakfast while lunch and dinner costs are borne by the player. While discussing the cost of spending, S1 also performs a metacognitive evaluation which is indicated by the statement "okay so per 477.000 person". Based on this statement, the thought of S1 is related to the opinion of the student with the initials C that the cost of Rp 4,770,000.00 represents the minimum expenses incurred by the team. Furthermore, S1 rethinks it by considering the absence of definite information about the cost of eating, then the proper solution should be to write down the minimum cost of expenditure issued by the team is Rp 4.770.000,00. Here are the results of interviews between researchers with subject S1 after discussion.

C: Okey is finished. 1090000 times 3 equals 3270000

S1: Continuous plus registration fee 1500000 so 4770000

Y: yes 4770.000 continue to be divided 10

S1: this is still not the cost of eating (metacognitive awareness).

C: food already include in hotel, hotel does not provide cook

S1: According to my experience, this lunch and its dinner not yet. (metacognitive evaluation)

C: does it need to be written at least so, so the minimum cost is 4.770.000

S1: Okay because it is not clear information about the cost of food, so the total expenditure of at least Rp 4.770.000,00

Figure 3 shows the result of "S1" after discussion.

   During the discussion, subject S1 also reconsidered how long the team will stay at the hotel. When reconsidering how many days the team will stay, the subject S1 performs a metacognitive evaluation activity marked by the statement "if 2 nights can be anyway, but the increment is expensive. So take 1 night aja. The 15th day comes, keeps going, the 16th after the game goes straight home ". Based on this statement, the thought of S1 is related to the question of a friend of S1 discussion with the initials C that the player only stayed overnight and then went straight home. As he goes on, S1 rethinks it by considering cost-saving expenses, he decides that the team is staying overnight. Here is the result of discussion between S1 with friends discussion in discussing the problem of stay time.
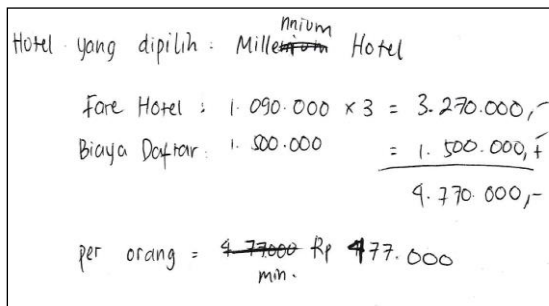
Figure 3. The Result of S1 After Discussion

C: Eh, .. that one night, then go straight home?

S1: Yes ... if in the hotel it's calculated per night. If 2 nights can still, but it will be expensive. So take 1 night aja. The 15th day comes, continues to stay, the 16th of it after a live battle home. (metacognitive evaluation)

Subject S1 revealed that to save expenses, better basketball team stay one night assuming, dated 15 juni teams have come in Jakarta and booking hotel to stay, then dated June 16 check out hotel and after finished the game they go home. Based on the idea of the S1 is finally all members of the group agreed that the team stay only overnight so that the total registration fee and hotel lodging of 4770 000 and the cost per player of 477,000. The Think Subject S1 structure when solving problems individually can be illustrated in Figure 4.
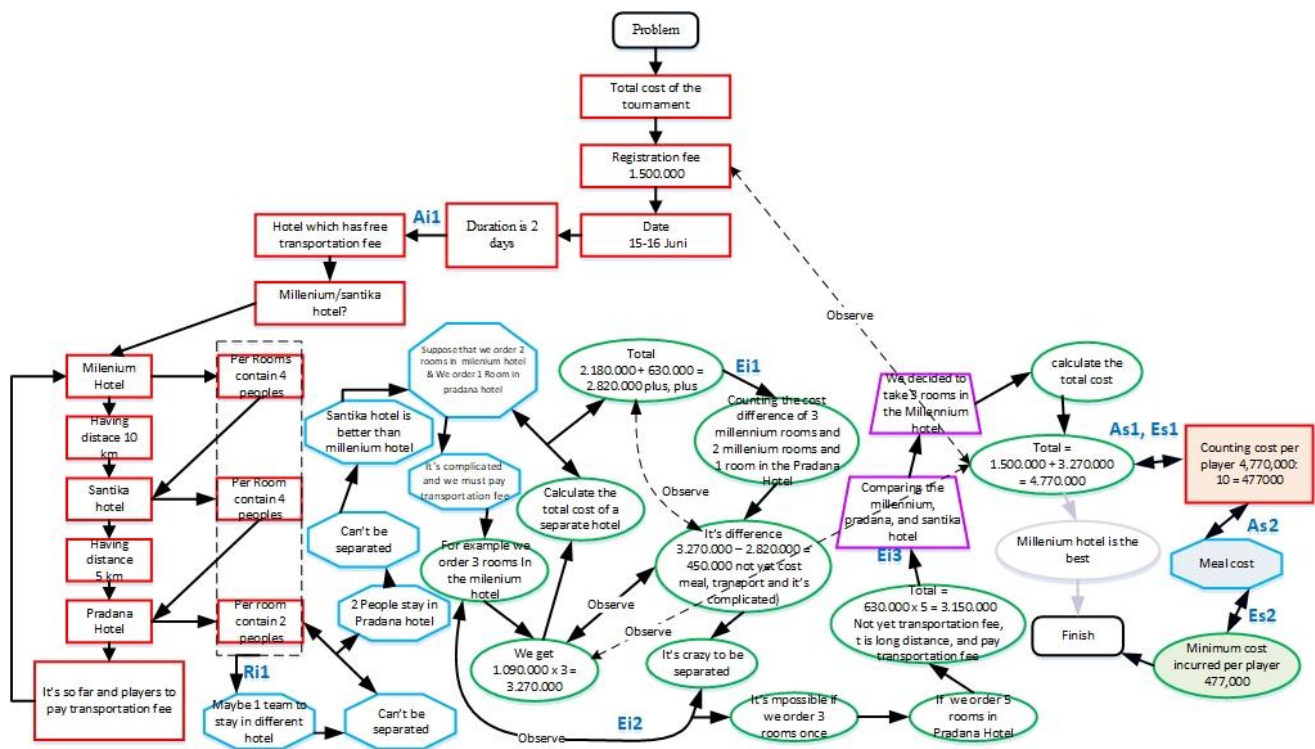

Figure 4. The Think Subject S1 structure when solving problems individually

Table 2. Caption Code Componen of Thinking Structure of S1 When Solving Problems in Groups

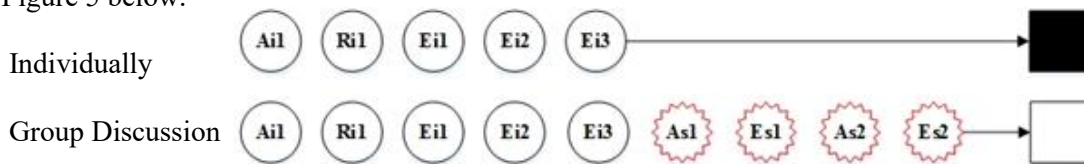| Code	Descriptions	Code	Descriptions
	Phase of understand the problem when solving the problem individually		Components of lost thinking structures
	Phase of devise a plan when solving problems individually	→	The order of activity when solving the problem
	Phase of carryout the plan when solving problems individually	Ai	Individual metacognitive awareness
	Phase of look back when solving problems individually	Ri	Individual metacognitive regulation
	Phase of understand the problem when gaining influence from group discussions	Ei	Individual metacognitive evaluation
	Phase of devise a plan when gaining influence from group discussions	As	Social metacognitive awareness
	Phase of carry out the plan when gaining influence from group discussions	Es	Social metacognitive evaluation

Based on the structure of the S1 thinking in Figure 2 when solving the problem individually and Figure 4 when solving the problem in group discussion, it can be grouped its metacognitive activity as in Figure 5 below.

Figure 5. The Shift of Metacognitive Activity Perspective of S1 Subject

Table 3. Notes of Code Components in Activities of Perfective Metacognitive Shifts of S1 Subject

Code	Descriptions	Code	Descriptions
○	metacognitive activity individually	■	Incorrect solutions
✷	Students' metacognitive activities in groups discussion	□	Correct solutions
A	metacognitive awareness	I	Individual
R	metacognitive regulations	S	Social
E	Metacognitive evaluation	1,2,3,...	Metacognitive activities in sequence

From Figure 5 above can be described that metacognitive activity S1 when solving problems individually consists of Ai1, Ri1, Ei1, Ei2, and Ei3. From these metacognitive activities, S1 produces a solution that the best hotel is Hotel Millennium with total cost incurred by players as much as Rp. 4.770.000,00. Solutions generated by the S2 are symbolized by a black box. Furthermore, during group discussion, there are 4 metacognitive activities of S1 symbolized by red color. The four metacognitive activities of S1 that occurred during the group discussion due to the influence of the discussion companion consisted of A1s, E1s, A2s, E2s.

Metacognitive activity begins when S1 goes into groups and discuss with his friends. The S1 subject gets feedback from his friend that he has not determined the cost per player. Based on input

from this friend, the subject S1 trying to understand and reevaluate what the problem so he realized that S1 forgot in determining the cost per player. Activity S1 in re-evaluating the problem and realizing that he forgot in determining the cost per player, showed that S1 did metacognitive awareness and metacognitive evaluation. From metacognitive awareness and metacognitive evaluation resulted in S1 realizing again that he has not calculated the cost per player and then he completes the answer again. Additional metacognitive activity in social also occurs during group discussions. S1 realized that he needed to re-plan the cost of eating that should be taken into account because the cost of eating in the cost of spending basketball players.

In planning again the cost of eating is a metacognitive awareness. Associated with the cost of eating that has been discussed, S1 discussion friend with initials C gave feedback that the last answer should be written down the minimum cost to be spent by the player of Rp. 477.000,00. Furthermore, S1 re-evaluates his friend's initials C that although the cost of eating is not mentioned in the issue, he needs to replenish his answer to a minimal cost that must be incurred by per player of Rp. 477.000,00. The evaluation done by S1 related to the cost of eating is a metacognitive evaluation.

Based on five individual metacognitive activities and four social metacognitive activities ie Ai1, Ri1, Ei1, E2, Es2, S1 subject produce solutions that the best hotel is Hotel Millennium with minimal cost incurred by per player of Rp . 477.000,00. Solutions generated by S1 when solving problems in groups are symbolized by a white box. The shift in metacognitive activity perfective S1 in question is a change in metacognitive activity of individual metacognitive activity (A1i, R1i, E1i, E2i, E3i) to social metacognitive activity (A1i, R1i, E1s, A2s, E2s) which Resulting in S2 retooling its thinking structure. Based on the change in metacognitive activity, S1 re-equip solution initially to be the best hotel is Hotel Millennium with total cost as much as Rp. 4.770.000,00 and minimal cost incurred by per player as much as Rp. 477.000,00.

4.3 Exposure Data of Subject S2 when Solving Individual Problems
At the stage of understanding the problem, the subject S2 performs a metacognitive awareness activity, visible from the thought of S2 is related to the question of the problem. Furthermore, S2 rethinks it by reading the problem so S2 realizes that the problem is a national basketball tournament held at Gelanggang Istora Jakarta on 15-16 June 2016, and 1 team consists of 10 people with registration fee Rp 1.500.000,00 . Furthermore S2 also performs another metacognitive awareness, visible from the thought of S2 is related to the registration fee. Next S2 rethinks it by not taking into account the registration fee, on the grounds that in determining the best hotel, the registration fee has no effect. This fact is seen from the results of think alouds S2 when solving problems and interviews conducted by researchers with subject S2 as follows.

S2: (Read instructions and problems). This year's National Basketball Tournament will be held at Gelora Senayan Stadium Jakarta on 15-16 June 2016. If you are the basketball team manager interested in joining this national basketball tournament, calculate the cost of each player as well as the total registration fee and the hotel be the best option for your team by considering all the data in the table. Your team consists of 9 players and 1 coach who is interested in joining the tournament, so there are a total of 10 people. Registration fee Rp. 1.500.000,00 per team. Hmmmm ... (stop any longer). (Re-read the problem) ... So the problem is a national basketball tournament held in the arena of 15-16June 2016. Total 10 people, cost 1500000. (metacognitive awareness). Hmmm ... means reply the cost of the enrollment is not my count because it is the same. Fixed costs, a total of 10 people (metacognitive awareness)

Further the results of think alouds above is reinforced by the results of interviews with the subject S2. The following is an excerpt of an interview between the researcher and the subject of S2
I : Okey what do you think after reading the problem from me?

S2: Previously, I think about the accommodation is also calculated what is not And the effectiveness like the hotel that can be seen from the facilities, distance, and free or not the cost of transportation. (metacognitive awareness).

Q: When you worked while I was voicing I heard my sister read this problem until many times, why dik?

S2: Because initially just read the description of the information contained in this table only and when read the first time still do not understand the essence like what

Figure 6 below shows the results of exploration S2 in understanding the problem.
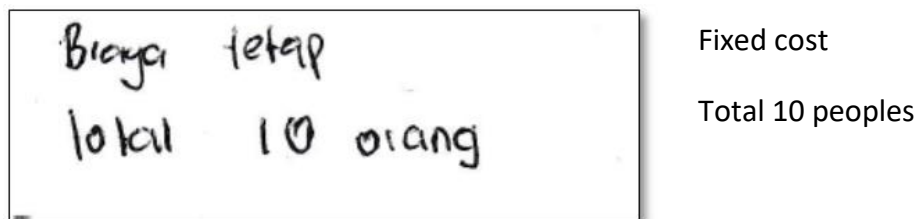
Figure 6. The Result Of Exploration S2 In Understanding The Problem

At the planning stage, the subject S2 doing metacognitive regulation activities are marked with the statement "You can let bandingin which is most effective so bu". Based on this statement, thought S2 is making a plan to be able to compare which hotels are more effective. Next S2 rethinks it by choosing a way to calculate the cost of lodging on each type of hotel based on the number of rooms needed. This fact arose from interviews between researchers and the subject of S2, and the following interviews.

I   : Why do you choose the way you write on this answer sheet?

S2 : the problem is to be able to compare which is most effective (metacognitive regulation)

I   : Okay, then how?

S2 : 4770000 this cost..emmm ... I used the accommodation for MilleniumHotel same registration fee of 1500000, then summed up and the total is 4770000 this.

At the stage of carrying out the plan, the subject S2 undertakes the plan made by calculating the cost of lodging on each type of hotel based on the number of rooms needed. First, if the team stay at Hotel Millennium then the number of rooms booked as many as 3 rooms because 1 team there are 10 people so the total cost of lodging is 3270000. Secondly, if the team stay at Pradana Hotel then the number of rooms booked as many as 5 rooms because 1 team there are 10 person so that the total cost of lodging is 3150000. Third, if the team stay at Hotel Santika then the number of rooms booked as many as 3 rooms because 1 team there are 10 people so the total cost of lodging amounted to 3960000. At the stage of implementing the plan, S1 conduct metacognitive evaluation activities, what S2 thinks is related to the cost of lodging in each type of hotel based on the number of rooms needed. Furthermore, S2 rethinks what has been thought of by reconsidering criteria at each hotel such as, many restaurants have little effect on basketball teams, and swimming pool facilities may be considered for physical training. Based on the considerations made, S2 decides the best millennium hotel for the basketball team. Here are the results of think alouds and interviews between researchers and S2.

S2: Millenium Hotel 1090000, 1090000, total 10 people, 3 rooms 3 million times, 90000 times 3 equals 270000, means 3270000 (free transport). Pradana hotel, it costs 630000 times 5 (while counting 63 times 5) 3150000, without transport, long distance, the hotel's restaurant is not

influential. Santika1320000 multiplied 3 (counting) 3940000, uh ... 3960000 transport, nearby, swimming pool facilities, basketball player does not impact but can make physical exercise. The best thing I think is the millennium, the best millennium (metacognitive evaluation).

The results of think alouds of S2 are reinforced by interviews as follows:

I: How does sister solve this problem?

S2: I was counting one of the Millenium Hotel. The price per night at MillenniumHotel 1090000 (while showing the results of his work) was made for 3 rooms, make 10 each play because the capacity per room only for 4 people. Keep this one (while pointing Pradana Hotel) that the capacity is only 2 people, the person is 10, this time the 5 is 3150000 but it is not pakek pakek transportation costs eh pakek transport fee, if the 3:200200 times 3 times this result (while showing the results of his work) although it is the most expensive but the closest too (metacognitive evaluation).

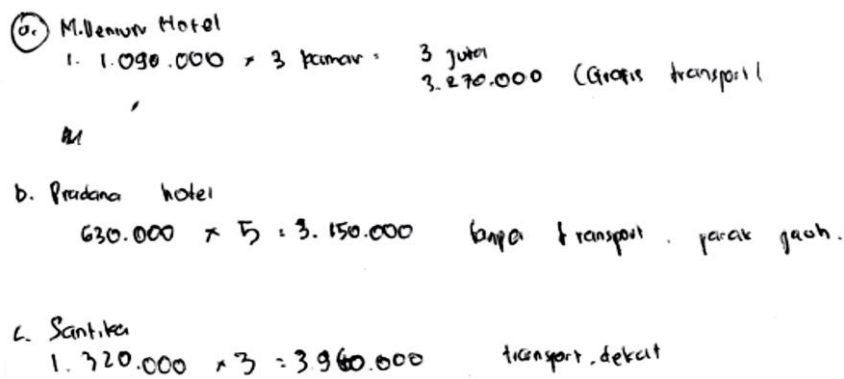Figure 7 shows the result of the S2 work while carry out the plan.

Figure 7. The Result Of The S2 Work While Carry Out The Plan

In the process of solving the problem, S2 performs the associated metacognitive evaluation activities in determining the cost per player. Metacognitive evaluation activity, seen from the thought of S2 is related to cost per player. Furthermore, S2 rethinks what is already thought (with regard to cost per player) that is by deciding that the cost per player is Rp 477.000,00, where the coach is also willing to pay. Here's the result of think alouds S2 in determining cost per player.

S2: so the cost per player means 3 million ... ohhh ... 3270000 plus 1500000. The total is 4770000dissed 10 because it's all paid for itself. Total 4770000, per player per player (silent long) .. emmm ... coach come pay or not? (silent long). Oiya coach follow pay. Accommodation costs per player when the coach comes to pay 477000, a total of 4770000 at Millennium Hotel (metacognitive evaluation).

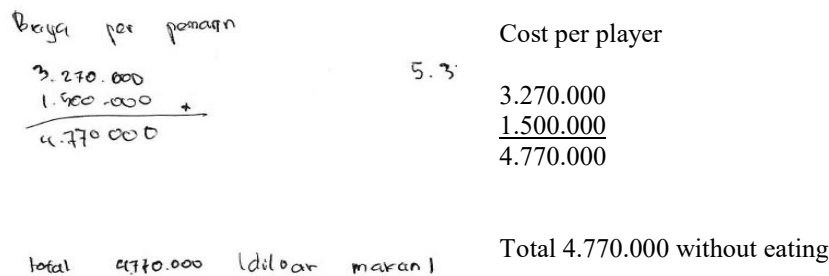Figure 8 shows the result of the S2 work in determining the cost of each player.

Cost per player

3.270.000
1.500.000
4.770.000

Total 4.770.000 without eating

Figure 8. The Result Of The S2 Work In Determining The Cost Of Each Player

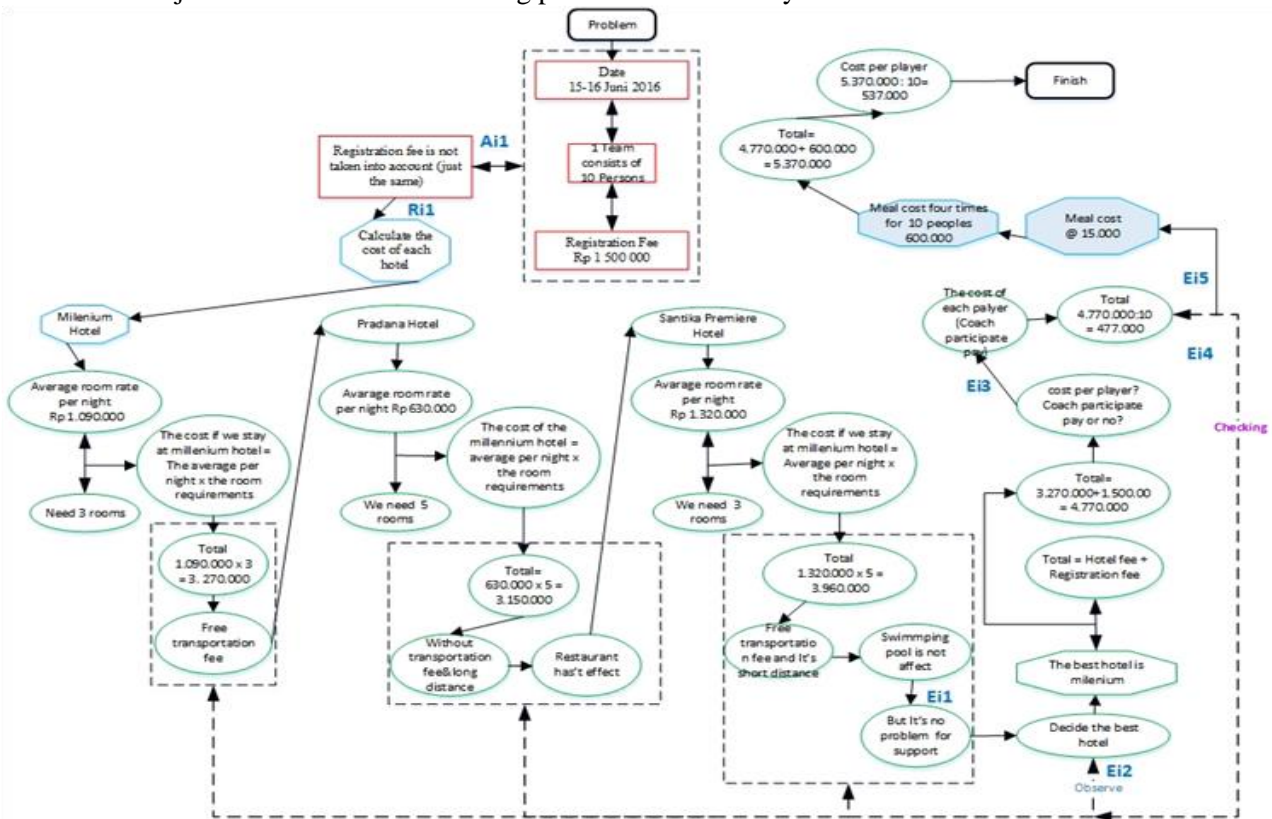Think Subject S2 structure when solving problems individually can be illustrated.

Figure 9. Thinking Structure of S2 When Solving Problems Individually

Table 4. Caption Code Componen of Thinking Structure of S2 When Solving Problems Individually

Code	Description	Code	Description
	Phase of understand the problem when solving the problem individually		The order of activity when solving the problem individually
	Phase of devise a plan when solving problems individually	Ai	Individual metacognitive awareness
	Phase of carryout the plan when solving problems individually	Ri	Individual metacognitive regulation
	Phase of look back when solving problems individually	Ei	Individual metacognitive evaluation

4.4 The Exposure Data of Subject S2 When Problems Solving in Groups

The first time during the discussion, S2 and 2 other friends discussed the cost of each player. They are looking for an agreement or a decision whether the coach is charged to pay or not. In seeking agreement to determine the cost per player, the subject S2 performs metacognitify awareness activities marked with the statement "Oiya, but I think 10 deh. sure? be borne by the players. become a burden for the players later. I mean already play, pay coach again ". Based on this statement, the thought of the subject of S2 is a question from a friend of the S2 discussion with initials R regarding cost per

player. Furthermore, S2 rethinks it, so S2 realizes that to determine the cost per player is the total registration fee and the hotel's lodging is divided by 10 This means that the coach will also bear the cost of his expenses so that will not burden the player. Here is the result of discussion between S2 and two friends
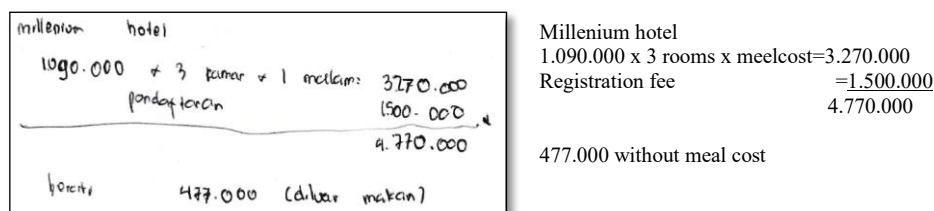
R : That means how many people will pay? 10 yes?
S2 : Oh yes, but I think 10 deh. do not need to be borne by the players. do not burden the players. The point is already playing, order pay coach again. (metacognitive awareness)
R : No information, meaning pay
S2 : if paid, it means the Millennium selected hotel

Figure 10 shows the result of S2 work after discussion.
.

Millenium hotel
1.090.000 x 3 rooms x meelcost=3.270.000
Registration fee   =1.500.000
                                   4.770.000

477.000 without meal cost

Figure 10. The Result of S2 Work After Discussion

During the group discussions, the subject of the thesis re-thought about the cost of eating. At the time of rethinking the cost of eating, the subject of S2 conducts awareness activities and metacognitif evaluation characterized by the statement "2 days with 4 meals, So 4 times 25000". Based on this statement, the thought of the S2 is related to the statement or feedback from a friend of S2 discussion with initials R that the cost of eating Rp 15.000,00 in Jakarta is too little. Furthermore, S2 reconsidered the matter by considering the opinion of his discussion friend R initials that the cost of one meal in Jakarta was Rp15.000,00 too small so that S2 decided that during the match in Jakarta, the cost of meals needed to be added again to Rp 25.000,00. In addition, S2 also considers that during a 2-day stay the basketball team needs to eat as much as 4 meals ie lunch and dinner assuming breakfast is usually provided by the hotel. Here is the result of discussion between the subject of S2 and two friends discussion.

S2: This meal has been provided ta?
K: I do not think so
R: In hotel there is not including ta meal?
S2: Yes, but sometimes tere is no breakfast
R: Oh yes. The most lunch in the game
S2: My meal counts once 15000
R: Assume that only 25000, and 15000 are too little
S2: Okay 25000 times 2
R: It ate of what day, 2 days?
S2: 2 days
K: How come just only, 2 days?
S2: 2 days with 4 meals, So 4 times 25000. (metacognitive evaluation)
R: 100000 is per person, means total cost incurred?
S2: 4777000 it makes if exclude food, if include food will be 5770000
R: Oh yes.
S2: Means 477000 same 577000

The results of these discussions were also reinforced by the results of interviews between researchers with S2. Here's the interview.

I : Okey after discussing with my friends just thinking back to the problem?
S2: So far the same but there is a change cuman changes in exclude eat tu plusin the cost of eating for 4 meals. (metacognitive evaluation)
I : Why do you think 4 meals are eaten?
S2: So, 2 days can be breakfast from the hotel. ie lunch and dinner for 2 days (metacognitive evaluation)
I : This is why you assume that the cost of food only 25000?
S2: Yes,.. only for one person, is enough to eat so,.. you know
Figure 11 shows the result of the "S2" work after discussion of the cost of each player.
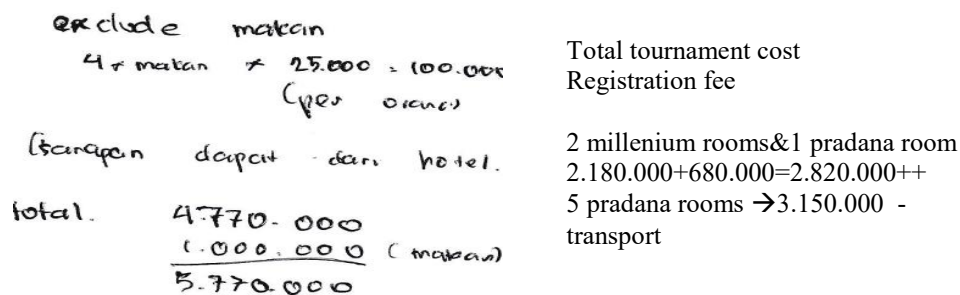

Figure 11. The Result of The "S2" Work After Discussion

Think Subject S2 structure when solving problems individually can be illustrated in Figure 12.

Based on the structure of the S2 thinking in Figure 9 when solving the problem individually and Figure 12 when solving the problem in group discussion, it can be grouped its metacognitive activity as in Figure 13 below.

Table 5 Notes of Code Components in Activities of Perfective Metacognitive Shifts of S2 Subject

Code	Descriptions	Code	Descriptions
⭘	metacognitive activity individually	▢	Incorrect solutions
✿	Students' metacognitive activities in groups discussion	▢	Correct solutions
A	metacognitive awareness	I	Individual
R	metacognitive regulations	S	Social
E	Metacognitive evaluation	1,2,3,...	Metacognitive activities in sequence

From Figure 13 above we can see that the metacognitive activity of S2 when solving individual problems consists of Ai1, Ri1, Ei1, Ei2, Ei1, Ei4, and Ei5. From these metacognitive activities, S2 resulted in a solution that the best hotel is the Millennium Hotel with a one-time cost of Rp.15.000,00 so the total cost incurred by each player is Rp.537.000,00. Solutions generated by the S2 are symbolized by a black box. Furthermore, during group discussion, there are 2 metacognitive activities S2 symbolized by red color. The two metacognitive activities of S2 that occurred during the group discussion due to the influence of the discussion companion consisted of A1s, and E2s.

The metacognitive activity begins when the S2 enters the group and discusses with a friend. S2 subject get feedback from his friend that the cost of one meal is Rp. 15.000,00 is still less so it needs to be added again to Rp 25.000,00. Based on input from this friend, the subject of S2 trying to realize that the cost of eating for Rp. 15.000,00 is too little and S2 reevaluates her friend's input that players
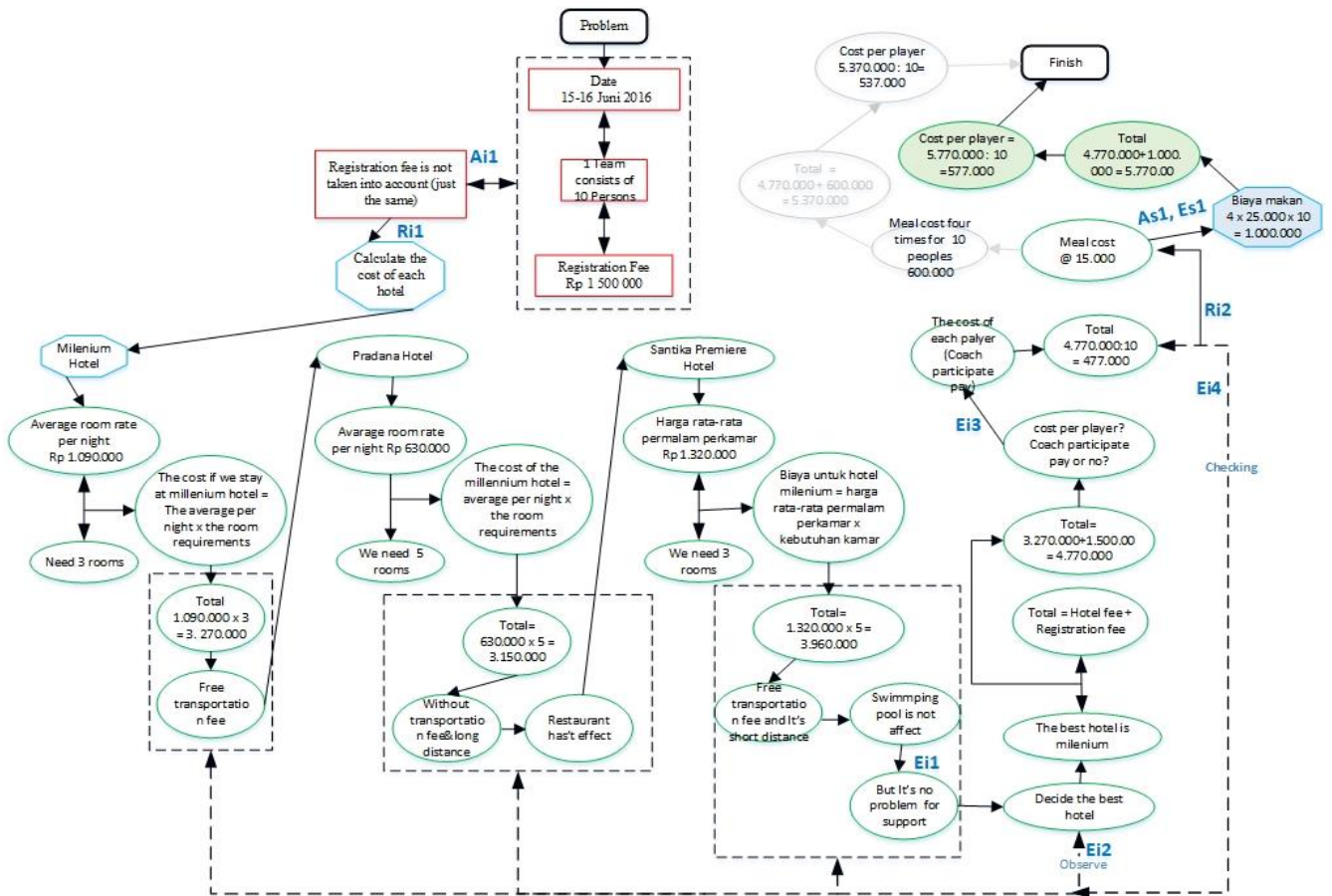
Figure 12. S2 Thinking Structure When Solving Problems In Groups

are competing in Jakarta so that the cost of food needs to be added again. Furthermore, S2 re-planned that the cost for one meal plus more to Rp 25.000,00 so the total cost incurred by per player amounted to Rp 577,000. S2 activity in realizing and reevaluating her friend's opinion until she re-planned the cost for one meal to Rp 25.000,00 is a metacognitive awareness activity and metacognitive evaluation.

Figure 13. The Shifts In Metacognitive Perfective Activities In S2 Subjects

Based on seven individual metacognitive activities and two social metacognitive activities is Ai1, Ri1, Ei1, Ei2, Ei1, Ei4, Ei5, As1, and Es2, the subject of S2 resulted in a solution that the best hotel is the Millennium Hotel at a minimal cost incurred by per player Rp. 577.000,00. Solutions generated by S2 when solving problems in groups are symbolized by a white box. The shift in metacognitive activity of the perfective S2 in question is a change in the metacognitive activity of the student from the individual metacognitive activity (Ai1, Ri1, Ei1, Ei2, Ei1, Ei4, and Ei5) to social metacognitive activity (Ai1, Ri1, Ei1, Ei2, Ei1, Ei4, Ei5 , As1, and Es2) which resulted in the S2 retooling its

thinking structure. Based on the change in metacognitive activity, S2 completes the original solution to be the best hotel is Hotel Millennium with total cost of Rp. 577.000,00.

The findings of the S1 who were influenced by group discussions related to cost per player are in line with one of the social-based characterizations proposed by Magiera & Zawojewski (2011). One of the social-based characterizations proposed by Magiera & Zawojewski (2011) is interpreting various perspectives. This characterization illustrates how one's thoughts are driven by another's mathematical approach, an example of considering new information generated by their peers and struggling to understand the mathematical explanations presented by others.

In relation to metacognitive activity undertaken by the subject in solving problems, Kim, Park, Moore, & Varma (2013) state that interaction with the learning environment, such as problem-solving activities and task complexity are the main sources that trigger metacognition. Furthermore, student interactions with learning environments such as group problem-solving potentially maximize opportunities for students to reexamine their thinking and improve their misconceptions.

The findings of the S2 who were influenced by group discussions related to the cost of eating in line with one of the social-based characterizations proposed by Magiera & Zawojewski (2011). One of the social-based characterizations proposed by Magiera & Zawojewski (2011) is to seek mathematical agreement. This characterization illustrates how the subject S2 seeks to reconcile the lack of agreement in the discussion process whereby this includes the subject of requesting the consent of the discussion friends regarding their interpretation of the problem situation.

5. Conclusion

The results of research on the shifting of metacognitive activity of students in solving mathematical problems consist of two shifts, they are the shift of metacognitive activity perfective and metacognitive activity constructive. The shift of metacognitive activity perfective occurs because of the influence of group discussions that result in students re-examining mathematical thinking. As students re-examine their mathematical thinking, there is a change of metacognitive activity of students from individual metacognitive activity to social metacognitive activity so that students retool their original thinking structures in mathematical problem solving

Research about the shift of metacognitive activity of students in solving mathematical problem is limited to the use of the same problem to be done both individually and group discussion, so researchers difficult to see the changes of metacognitive activity that occurs from understanding problems, planning, implementing, and looking back. Based on this description, the researcher provides suggestions for further research on the shift in metacognitive activity of students using similar problems in order to see the changes in each metacognitive activity taking place.

Based on the results of the research, the researcher suggests that the learning process related to solving mathematical problems needs to be designed in the form of group discussion, so that the students are trained to re-examine their mathematical thinking and the problems they face can be solved.

References

[1]   Aurah C, Keaikitse S, Isaacs C, and Fincii H 2011 The Role Of Metacognition In Everyday Problem Solving Among Primary Students In Kenya *Problems Of Education In The 21' century* **30 (2)** 9-21

[2]   Chiu M and Kuo S 2010 From Metacognition to Social Meta cognition: Similarities, Differences, and Learning *Journal of Education Research* **3 (4)** 321-338

[3]   Goos M 2002 Understanding Metacognitive Failure *Journal Of Mathematical Behaviour* **21 (3)**

283-30

[4]   Goos M, Galbraith P, and Renshaw P 2002 Socially Mediated Metacognition: Creating Collaborative Zones Of Proximal Development in Small Group Problem Solving *Educational Studies in Mathematics* **49 (2)** 193-223

[5]   Hastuti I D, Nusantara T, Subanji, and Susanto H 2016 Constructive Metacognitive Activity Shift In Mathematical Problem Solving *Educational Research and Reviews* **11 (8)** 656-667

[6]   Hurme T, Marenluoto K, and Jarvela S 2009 Socially Shared Metacognition of Pre-Service Primary Teachers in a Computer-Supported Mathematics Course and Their Feelings of Task Difficulty: a Case Study *Educational Research and Evaluation* **15 (5)** 503-524

[7]   Kapa E 2002 A Metacognitive Support During The Process of Problem Solving in a Computerized Environment *Educational Studies in Mathematics* **47 (3)** 317-336

[8]   Kim Y, Park M, Moore T and Varma S 2013 Multiple Levels of Metacognition And Their Elicitation Through Complex Problem-Solving Tasks *Journal of Mathematical Behavior* **32 (2)** 377-396

[9]   King F, Goodson L, and Rohani F 1993 *Higher Order Thinking Skills* (New York: Assessment & Evaluation Educational Services Program)

[10]   Kuzle A 2013 Patterns of Metacognitive Behavior During Mathematics Problem-Solving in a Dynamic Geometry Environment *International Electronic Journal of Mathematics Education* **8 (1)** 20-40

[11]   Magiera M and Zawojewski J 2011 Characterizations of Social-Based and Self-Based Contexts Associated With Students' Awareness, Evaluation, and Regulation of Their Thinking During Small-Group Mathematical Modeling *Journal for Research in Mathematics Education* **42 (5)** 486-520

[12]   Mokos E and Kafoussi S 2013 Elementary Students' Spontane ous Metacognitive Functions in Different Types of Mathematical Problems *Journal of Research in Mathematics Education* **2 (2)** 242-267

[13]   Wilson J and Clarke D 2002 Monitoring Mathematical Metacognition *Paper presented at the Annual Meeting of the American Education Research Association* (New Orleans: ReportsResearch)