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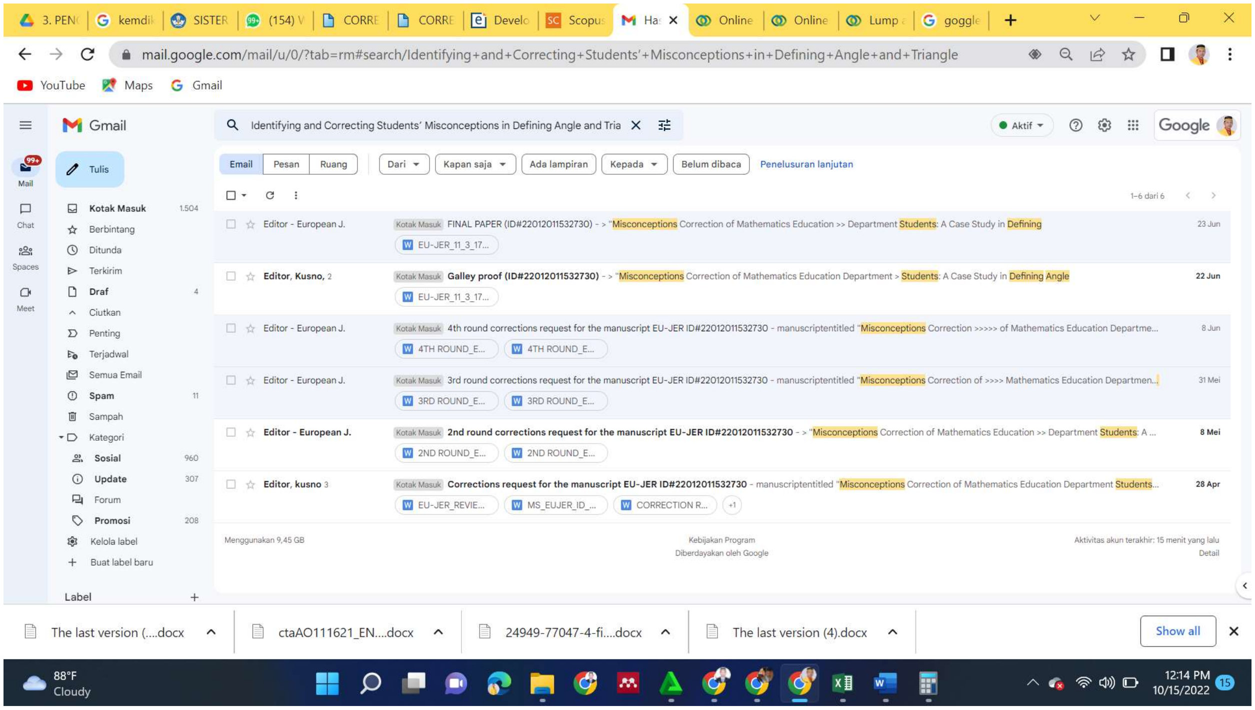
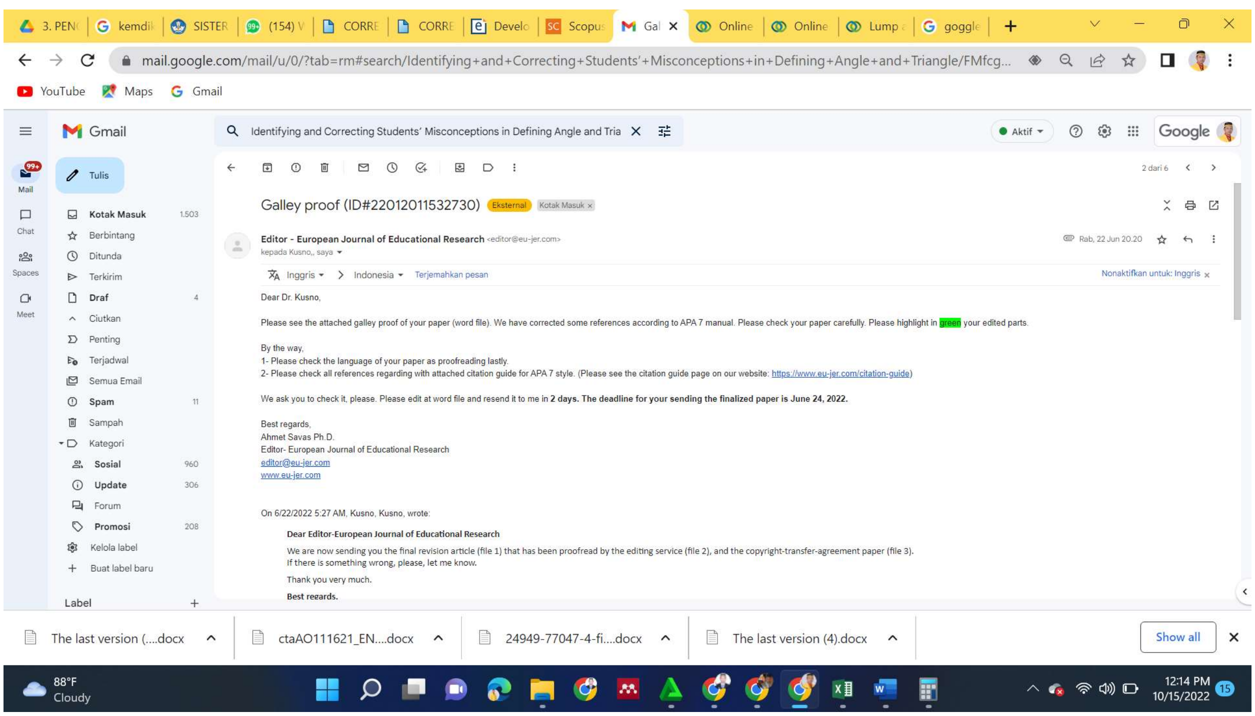
Identifying and Correcting Students' Misconceptions in Defining Angle and Triangle





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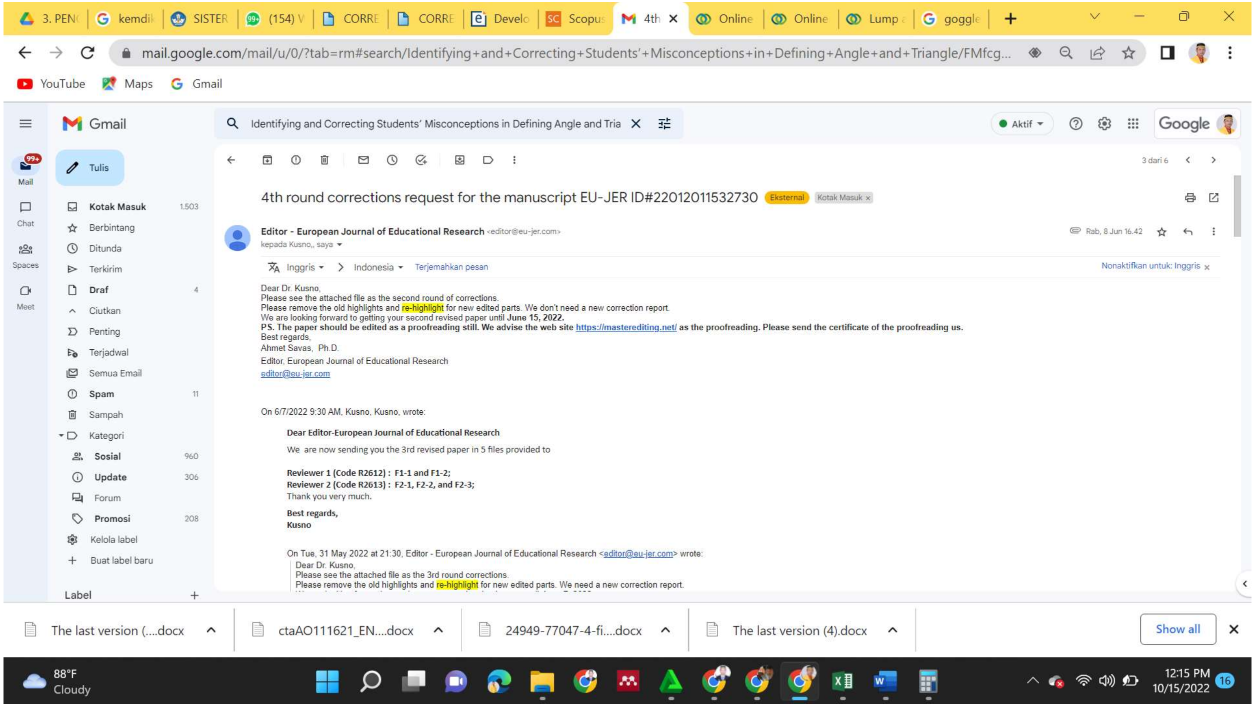

4th round corrections request for the manuscript EU-JER ID#22012011532730

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Dear Dr. Kusno,
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We are looking forward to getting your second revised paper until **June 15, 2022.**
PS. The paper should be edited as a proofreading still. We advise the web site https://masterediting.net/ as the proofreading. Please send the certificate of the proofreading us.
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On 6/7/2022 9:30 AM, Kusno, Kusno, wrote:

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Reviewer 1 (Code R2612) : F1-1 and F1-2;
Reviewer 2 (Code R2613) : F2-1, F2-2, and F2-3;
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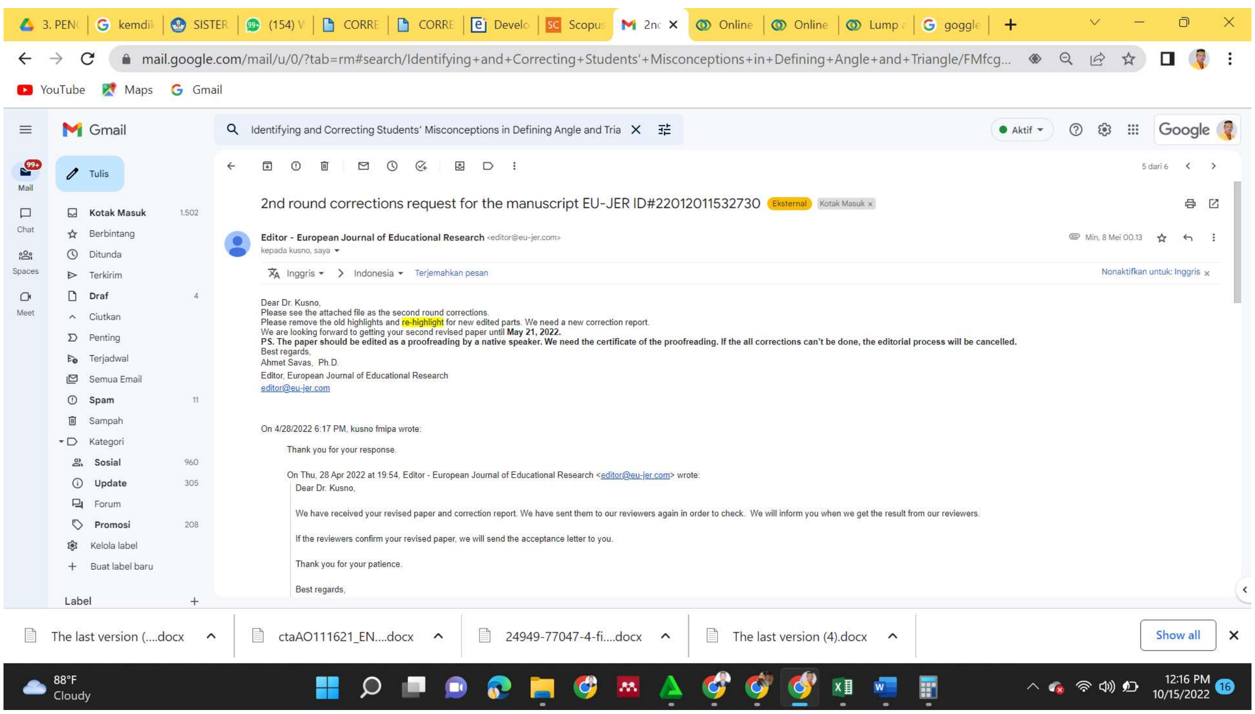

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On 4/28/2022 6:17 PM, kusno fmipa wrote:

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Review Form

Manuscript ID:	EU-JER_ID#_22012011532730	Date: 7 May 2022
Manuscript Title:	Misconceptions Correction of Mathematics Education Department Students: A Case Study in Defining Angle and Triangle	

ABOUT MANUSCRIPT (Mark with "X" one of the options)	Accept	Weak	Refuse	Not Available
Language is clear and correct		X		
Literature is well written		X		
References are cited as directed by APA	X			
The research topic is significant to the field	X			
The article is complete, well organized, and written		X		
The research design and method are appropriate		X		
Analyses are appropriate to the research question		X		
Results are presented		X		
A reasonable discussion of the results is presented		X		
Conclusions are clearly stated		X		
Recommendations are clearly stated		X		

GENERAL REMARKS AND RECOMMENDATIONS TO THE AUTHOR

First of all, the authors' responses to my comments are not clear and also satisfactory. My specific comments about the revised paper are below.

Title
The title does not make sense to readers. Please change it.

Introduction
The introduction is not still explaining the rationality based on the literature. The authors do not present any information about the existing literature. They need to emphasize the research gaps in the literature. Mainly, there is a still need to emphasize the weakness and strengths of the existing studies on geometry, misconceptions, and the MDWI.

What is the MDWI? I could not still find its full name in the text.

Literature Review
The literature review is not a real review of the literature. The authors do not present the main results of previous studies about geometry, misconceptions, and the MDWI. After presenting the main results of the research studies, they need to make an overview of the existing literature.

Problems and Purposes of Research
What are the results from studies about the angles and the triangles? What do we know about geometry, misconceptions, and the MDWI? What do not we know about these dependent variables?

Framework of Research
This section is still too general and also overly long. If the aim is to explain the implementation of the MDWI approach, then the information given in this section should be moved to the method. I could not

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understand what the authors sought to mention.

Method
The authors need to answer the following questions.

How the instrument was developed? How the validity and reliability were ensured?

How the data were analyzed? There are no details about this detail.

My major concern is that it is not clear how the MDWI was implemented during the data collection?

Discussion
The results should not be repeated in the discussion. The main results should be given briefly. The authors need to discuss possible reasons for the results should be discussed.

Conclusion
What is the contribution of this study to the literature? What is the new knowledge of the literature?

Recommendations
Implications for further studies are not satisfactory. Please explain what should be researched specifically by scholars in the future?

Language
The language is not still acceptable. Proofreading by a native speaker is mandatory. Otherwise the paper will be refused.

Overall, I think the revised paper needs some major revisions.

THE DECISION (Mark with "X" one of the options)	
Accepted: Correction not required	
Accepted: Minor correction required	
Conditionally Accepted: Major Correction Required **(Need the second review after corrections)**	X
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Reviewer Code: R2612 (The name of the referee is hidden because of blind review)	

Misconceptions Correction of Mathematics Education Department Students: A Case Study in Defining Angle and Triangle

Abstract: Misconceptions are one of the most handicaps to comprehend in learning mathematics. This study aimed to investigate the common errors and the misconception causes of students in defining the angle and triangle. Moreover, the research introduces the metacognition regulation, drawing geometry concepts, students' writing tasks, and lecturers' intervention effort with the social involvement, called (MDWI) approach to exchange the understanding from the wrong concepts to be the correct concepts of geometry ideas. To achieve this goal, it used a research design. It identified and resolved the errors in defining the angle and triangle of the first-year undergraduate students of the mathematics education department from an excellent private university in Mataram, Indonesia. The steps were as follows. Using open-ended questions tests and in-depth interviews, we identified the errors, the roots, and the causes of learners' misunderstanding. Then, implementing the MDWI treatment introduced the way to correct these errors correction. The research found that learners generally fail in concept images interpretation, logical thinking, and knowledge connection needed to define the angle and triangle. The MDWI approach remedied the misconceptions of generalization, concept images errors, and connection incompetencies of geometry features.

Keywords: *Angle and triangle, cause, misconception correction, common errors, treatment.*

Introduction

It reports that the ability of school students is still low in Indonesia. The results of tests and evaluations from the International Student Assessment (PISA) along the years 2015 and 2016 undertaken by the Organization for Economic Cooperation and Development are classified as low completeness (OECD, 2016). The tested material also includes geometry. It means that students have not fully mastered geometry's main subjects, including quantity, relationship, and uncertainty (Lemke et al., 2004). In 2019, the mathematics score's national exam of junior high school students failed, namely in the position 45 of score interval 0-100. Besides, for senior high school level, the student's achievement to answer all given questions of the geometry and trigonometry test with correct responses category was only 37% in 2017; and 34% in 2018. Mastering the learners' mathematics was still not good, i.e., in average score 45 of interval 0-100 (Kemendikbud, 2019). These fail cases will potentially be repeated by the school students in future studies, particularly in the first year of an undergraduate program.

The learners' weak understanding of geometry definitions and concepts as the geometry pre-knowledge will affect low future mastering geometry, difficulties, and failure. One factor in which the students can happen is the geometry misconception when their concentration only focuses on the physical shape and the geometry images rather than identifying the essential geometry properties of represented figures (Biber et al., 2013; Poon & Leun, 2016). Based on the geometry questions to the students about the definitions of angle, measure, and shape, this study reported a lack of students' background knowledge that makes many learner misconceptions in reasoning and basic operation mistakes (Özerem, 2012). These facts

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indicate that the learners have to develop their understanding of geometry concepts and some related abilities.

A theoretical and passive teaching approach, be likely to memorize, and the few visuals provided do not guarantee that students can master the definitions and geometry concepts. Stimulating and challenging geometry concepts is required that students will be able to improve their understanding. Several result studies informed that students failed to complete dimensional deconstruction of images to obtain mathematical properties. They found it difficult to determine the characteristics of figural elements relevant to the concept (Cunningham & Roberts, 2010; Gal & Linchevski, 2010). This misunderstanding can occur because of the teachers themselves or their environment, i.e., the proficiency and inadequate book facilities. Although the teachers' role in developing knowledge is essential, they also need to play an active role in dealing with their misunderstanding problems. Through metacognitive activities, such as their own written work error analysis, students can find and try to align their conceptual inconsistencies with more formally accepted mathematical constructs (Tirosh, as cited in Kembitzky, 2009, p.2). This conceptual change needs some strategies and metacognitive skills. Referring to Stepans' model of conceptual change, to change the alternative concepts of students needs the nature of learning tasks that can help learners exchange their understandings with the right ideas. The nature of the learning environment can involve social dialogs and negotiations among learners (Sarar & Al-Migdady, 2014). Because of these geometry learning impediments, we have to make aware and strengthen the conceptual understanding in the error correction of geometry lessons. Consequently, it needs a new learning treatment approach to improve the correct geometry concepts and reasoning. For this reason, we will evaluate the students' error correction results in the teaching-learning process. In short, this research will apply the MDWI teaching-learning method for exchanging the students' concepts errors to be the correct concepts.

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Literature Review

Metacognition is vital in supporting the performance of cognitive tasks in mathematics learning. Some research results reported that metacognitive understanding includes aspects of cognition's knowledge and cognition's regulation. This knowledge contains the cognitive abilities, processes, resources, and the influence of a person, task, or strategy factors on performance (Brown et al., as cited in Garofalo & Lester, 1985, p.164). The regulation of metacognition is concerned with the strategic decisions' activities in a course via cognitive tasks. These activities consist of planning studies, monitoring processes, evaluating and revising the outcomes. Robert J. Sternberg (in Artzt & Thomas, 1998) stated that metacognition is various. It includes both understanding and control of cognitive processes that include planning, monitoring, and evaluating activities. This understanding process, of course, must be effective action. It also must be remembered that metacognition interacts with many other aspects of the student, i.e., abilities, personality, and learning styles. Magiera and Zawojewski (2011) used metacognitive awareness, regulatory, and evaluative approaches for small-group learners. They identified and characterized the social-based and self-based contexts related to their metacognitive activities in mathematical modeling learning. Based on these research studies, the metacognition approach supports the learning process in mastering concepts and geometric reasoning. The application of metacognition regulation expects to prevent students from thinking that they only memorize concepts. It can be a tool to make learners aware and correct their reasoning weaknesses in mastering the concept of geometry. In the learning process, the teacher can use it to determine the weak points of student reasoning, develop the strategies for learning, and determine some stages of student error correction.

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Visual geometry objects help build people's experiences, beliefs, and understanding of an item through a cognitive process. Logically, the images of geometry objects are useable to explain the relationship between one and another concept. The efficacy of visualization will help understand concepts and support learners to acquire educationally required knowledge (Phillips et al., 2010). On the other sides, as a tool in learning geometry, images of geometry objects are widely used to explain definitions and concepts of geometry, but some learners still have misconceptions. Students usually know the ideas from their memory associated with the figures. Unfortunately, they fail to match both concept's formal definition and the geometrical figures (Vinner & Hershkowitz, 1980). Berthelot & Copy (in Poon & Leun, 2016) stated that one of the students' misconceptions factors in geometry learning is an incapability to identify various shapes (symbolic, visual, etc.) of the same geometry concept. Özerem (2012) found the student feebleness of measures, angles and shapes, transformations and construction, and 3-D shapes.

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Many misconceptions students also reported the lacking of background knowledge, reasoning, and basic operation mistakes. Hence, applying geometry images to aid the learners in understanding abstract geometry ideas, the teachers need to identify, verify, and evaluate the understanding errors and misconceptions of ideas. Harmonizing the concept images seen and what students have in mind can use some geometry pictures to develop cognitive conflict strategies and counterexamples for the learners. In learning, understanding a geometry concept of the students can have misconceptions, namely different from her/his accepted ideas. It is a real challenge for university educators in which they generally refuse to be corrected. Therefore, providing personalized interventions to help learners resolve misunderstandings in this context is a difficult challenge. Educators must work with their learners to identify, recognize, and correct commonly held misconceptions to attain the best learning outcomes. Any student misconceptions critically need to be evaluated, revised, and changed with information consistent with the accepted concepts (Verkade et al., 2016). Generally, there were five causes of errors: language misconceptions, spatial information difficulties, deficient mastery of prerequisite skills, facts, and concepts; fallacies of thinking; and the application of irrelevant rules or strategies (Radatz, 1979 as cited in Kim, 2011). Ay (2017) reviewed the errors. It states that apart from the test, one of the most appropriate ways for detecting these students' misconceptions is collecting qualitative data through interviews or observations. This data can provide in-depth information about students' knowledge. However, the researchers did not prefer going on further steps. Only a few studies applied a treatment to see whether the therapy could eliminate students' present misconceptions.

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Teacher's teaching and intervention strategy can make a difference in students' comprehension, which is essential in instructional practice and student learning. In geometry learning, Lim state that the information's communication at the different level of reasoning among the teacher and student becomes a common cause of misconception. When teachers explain different geometry thinking levels to learners, the concepts are not fully understood or acquired. Teachers must know their students' level of geometrical understanding (in Luneta, 2015). Battista et al. (1991) reported that developing the students' meaningful comprehension of geometry concepts requires an appropriate instructional task and assessment in teaching and learning geometry. Clarke et al. found that writing allows a teacher to see the kind of thinking and understanding that is not easy and accessible via the computational and proficiency test. Teachers can examine the process of sense-making when students explore and work with mathematics (in Kembitzky, 2009). Therefore, teacher intervention and student writing assignments will direct the achievement of conceptual understanding following curriculum objectives. It can help the learners to use previous experiences correctly and providing a new comprehension of the shortcomings of prerequisite material that students do not yet have and avoiding understanding concepts via rote, but by understanding processes.

The students who recognize misconceptions of geometry concepts will open their awareness of striving to improve themselves and encourages mathematics educators to develop effective strategies in helping the learners to correct the wrong ideas in the right way. However, the studies for correcting these misunderstandings in defining geometry concepts are rarely resolved. For this reason, the discussion in this paper focuses on finding common mistakes of mathematics education students about concepts of angles and triangles. Then, applying the MDWI approach would undertake misconceptions correction of the learners, i.e., using a metacognition regulation (M) and students' actions to draw all figural concepts (D) to identify the pictures' figural elements relevant to the angle and triangle concept. Then, they write the ideas (W) in definition form aided by the intervention and instruction of the lecturer (I).

Problems and Purposes of Research

Students' ability to define geometry concepts is one of the main goals of achieving the first-year undergraduate program's mathematics competencies. Unfortunately, using geometry figures and tools to support the learners in understanding the concepts and the definitions invoke some errors and misconceptions. They are difficult to determine the characteristics of figural elements relevant to the idea and often fail to arrange the words to construct the alternative definitions. To exchange the learners' incorrect concepts with the right ideas, of course, needs the nature and suitable learning methods. This research addressed to answer the following problems.

1. What do the common errors and the roots of misconceptions often happen in defining the angles and the triangles of mathematics education students of the undergraduate program?
2. How can MDWI resolve and exchange the students' misconceptions about definitions of angle and triangle from misconception concepts to be correct concepts in the teaching-learning process?

This research aimed to investigate the students' common errors and the causes of misconceptions in defining the angle and the triangle. Moreover, it introduced the benefits of MDWI approach for exchange the students' understanding from the wrong concepts to be the correct concepts about angle and triangle.

Framework of Research

Stage 1: Identification of Misconception

Students' misconceptions of geometry concepts can occur due to many factors such as student experiences and learning approach, teacher roles, and facilities (Cunningham & Roberts, 2010; Gal & Linchevski, 2010; Özerem, 2012; Poon & Leun, 2016). In this case, if the student's previous knowledge is likely to be lacking or there is a conflict with something learned, then the student's misconceptions need to be replaced. On the other hand, students' previous knowledge of a concept is correct, whereas understanding a new idea is not perfect, necessary to fill the understanding gap with authentic experience. Hence, determining the strategy and treating process relevant to students' misconceptions needs to identify the students' misconceptions types. From the introduced researches, it was founded some student misconceptions in geometry learning. For example, the student doesn't know that the rotation aspects contain: the direction, preserving parallel, and lengths. Some learners have errors relating to a deficiency of understanding geometry figures and insufficient knowledge of proof's importance. They also fail to state the particular polygons and their features of these polygons (Alamian et al., 2020; Cirillo & Hummer, 2019; Herholdt & Sapire, 2014; Junus, 2018).

Stage 2: Diagnosis to Find the Roots and the Causes of Problems

The purpose of students' misconceptions diagnostic is to identify their error levels and find out the causes and the roots of their difficulties in teaching-learning geometry. Moreover, it uses to investigate the weaknesses of the teaching approach and the facilities used by educators. Using the interview method for diagnosing the errors can provide recent information of student weaknesses and flexibility of examining; meanwhile, open-ended test methods will support the students' chance to write their answers in their own words. They will probably give some new valuable responses (Gurel & Eryılmaz, 2015). This research categorized the misconceptions diagnostic of students into the following levels: almost understanding, inadequate understanding with minor and major misconceptions. Some research studies reported the concepts misunderstanding's causes that should erase. Because mathematical materials are generally interconnected, the students' misconceptions in previously discussed topics should eliminate before introducing a new issue (Ozkan et al., 2018). Al-Khateeb (2016) also states that one of the misconceptions causes is the lack of prior knowledge and insufficient students' knowledge of the geometry concept. Regarding these studies, evaluating the students' geometry pre-knowledge and inter-connecting their geometry concepts and ideas is essential to find their trouble roots.

Stage 3: Strategy and Correction of Misconceptions Using MDWI

The strategy and errors corrections use three treatment steps, i.e., student awareness; defragmentation, reconstruction, and geometry concepts connection of learners' knowledge; revision and decided to exchange from the wrong to the right ideas. The ways are as follows.

Step 1: Student Awareness of Errors and Difficulties

The first step to change the misconception is to make the student aware that there is an error from the beginning. Removing the students' impediments must crucially come from themselves, including the held beliefs and prior knowledge (A Handbook for Educators, 2016). Kruger and Dunning (2009) also warned that learners who do not know their abilities would suffer a double burden; first, they only reach wrong conclusions; second, they are problematic to raise metacognitive skills to realize it. A study by Taylor and Kowalski (2004) informed that the power of belief is a significant transitional variable that may turn over in one's mind the change process. Furthermore, Hughes et al. (2013) concluded that it is easier for learners to disregard, reinterpret, or refuse new information rather than to change their beliefs. Conceptual change needs the conditions that students must be motivated, i.e., unsatisfied with their previous view. The alternative explanatory concept must be clear, i.e., helpful and coherent. Then, this substitute concept must be reasonable, i.e., rational to the learners. Finally, the substitute idea must be valuable, i.e., helps the learners in resolving problems.

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Step 2: Defragmentation, Reconstruction, and Connection Treatments of Incorrect Concept

- *Metacognitive Regulation (M)*

Supporting and guiding student errors correction is needed via the learning process. Applying the MDWI with the metacognitive processes approach: planning strategies, monitoring, and evaluation (Artzt & Thomas, 1998; Garofalo et al., 1985) will expect to resolve the students' wrong concepts through defragmentation or substitution, reconstruction, and connection processes of thinking structures. These processes are, respectively, to reconstruct thinking fragmentations of misconception, link knowledge and rectify an idea error minor, rearrange their knowledge structures and the logical thinking error. In this paper, we condition the

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Commented [A12]: Invalid "et al." (Two authors) students to be aware of the troubles as they know them from the first place, and then, the educator brings the learners to be active thinking to exchange the wrong concept with the right ideas. The metacognitive regulation guides the learners and instructors to design, control, evaluate, correct the understanding of geometry al features and structure the words for defining angles and triangles.

For learner errors treatment, the scheme of metacognitive regulation involves the students' understanding of the geometry pre-knowledge for constructing an angle and triangle, i.e., point, line, position, and direction. Using undefine terms, the learners must recognize the formal definitions of a line segment and a ray. Drawing, connecting, or combining among the points, the lines, the segments, and the rays, they have to try to find the shapes of the angle and the triangle. Using these constructed pictures, the students practice to explain and compose the definitions with their words. In short (Figure 1), they must resolve four treatment stages in the process of correcting misconceptions, i.e., recognizing primitive concepts and adding their pre-knowledge related to the defined geometry concepts (M1); drawing and demonstrating the concepts' figures (M2); presenting and writing geometry ideas connected with the prior knowledge (M3); composing the formal definitions (M4). These treatment series help the learners to identify, reflect, evaluate, and correct the misconceptions of angle definition and triangle in natural ways. From the teacher's side, it can quickly detect and correct the students' errors from these stages. Thus teachers' instruction and intervention for student error correction will be more focused and effective.

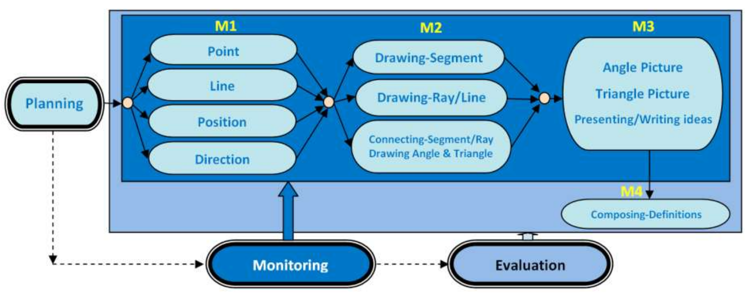

Figure 1. Metacognitive Regulation Scheme

- *Drawing Geometry Figures Based on Recognizing Primitives Concepts (D)*

The use of figural and visual representations of a geometry concept is to help students understand the abstract ideas; however, there are many misconception cases due to lack of prior knowledge related to these geometry concepts development (Battista et al., 1991; Özerem, 2012). The learners were also making errors evoked by the concept image, i.e., the set of all the mental pictures associated in the student's mind with the concept name, together with all the properties characterizing them (Şahin et al., 2020). Consequently, these students fail in the construction of the definitions and misunderstanding the mathematical concepts. When the students' knowledge of geometry properties is incomplete, their concept image will differ from the required formal definition (Kembitzky, 2009; Poon & Leun, 2016). Thus, students' thinking approaches in defining a geometry concept should be emphasized to avoid memorizing image concepts. It also suggests that through a reflection and understanding guided by the educator, they should strictly depart from undefined geometry terms (primitive) and drawing geometry figures experiences to build a geometry concept in the standard definition form and other geometry terms in the broad sense.

- *Writing Task to Present Ideas and Compose Definitions (W)*

Students' writing use to stimulate the dialogue for direct and indirect communication between the learners and the teacher in the teaching-learning process. Pugalee confirmed that writing helps mathematical thinking and supports learners in internalizing them of productive communication and relationship (in Urquhart, 2009). Through students writing, it could be known and assessed for the correctness of their already acquired mathematical. The studies discovered that the student's understanding levels improved, exchanged the reflections, and re-evaluated their answers. They are more developed in the competencies of reasoning, and ideas. They also improve to links between abstract mathematics and the context questioned. (Barbara et al., 2016; Freeman et al., 2016; Wilson & Nebraska, 2009). In short, it showed that there was a change from passive to active learners and the improvement of mathematical reasoning skills (Edig & Chavez, 2017).

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- *Interventing Students with Social Involvement (I)*

One of the most common conceptual changes using intervention and instruction strategies in the classroom was to induce cognitive conflict through presenting unusual (odd) facts or contradictory information (Limo´n, 2001). The mental conflict's primary goal is to make the learners disappointed with their present conception (Ozdemir & Clark in Kabaca et al., 2011). The educator can apply these strategies for students in both a lack of prior knowledge (missing knowledge) and the existence of incomplete knowledge or knowledge gap Chi, 2008; Chi, 2013). On the other side, Kowalski and Taylor's study suggested that the educator implement a critical thinking method to predict student misconceptions changing. Changing the students' misconceptions can happen for any abilities level and, significantly, to correct learners who think critically (Kowalski & Taylor, 2004). Using small group discussion, the educator may use Stepans' model for aiding students thinking contrary to their existing ideas. Through this group, the learners adjust to the new concept and resolve any current contradictions. Then they develop the concepts by connecting the thought learned in class with other associated concepts and ideas (in Sarar & Al-Migdady, 2014). In this action, the top priority of the educator's interventions and challenges is how to connect and correct the previously learned material of the student with their new knowledge.

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- *Summary*

The MDWI approach aims to help the learners to exchange errors concepts with the right ideas. This approach, principally, encourages students to revise their existing preconception errors and accommodate the new idea. Hopefully, it can give learners opportunities to reflect and negotiate true mathematical meaning with their misconceptions and help them become stronger learners. They become much better prepared to exchange their incorrect geometry al concepts with correct concepts and proper decisions. Applying this MDWI strategy, we defragment and reconstruct or substitute the student's misconception ideas and an understanding gap with natural ways and considering of following aspects. It involves the motivation and beliefs, prior knowledge, and cognitive engagement related to students; content knowledge, interests, and teaching strategies related to educator; the role of peers learning and learners-educator relation associated with the social context. The students' eliminated errors correction includes misconception of generalization, concepts images, geometry features, and properties or others (Figure 2).

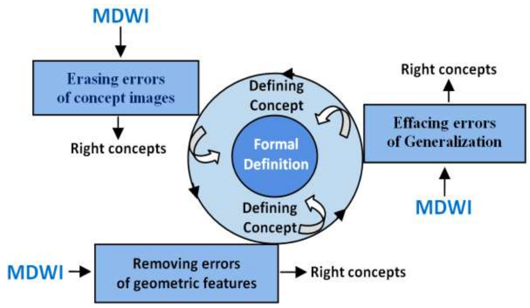

Figure 2. Errors Elimination

Step 3: Decision of Incorrect Concepts Exchange with Right Concepts
After MDWI treatment in Step 2, we evaluate the action results. When the student's works are considered satisfactory and coherent with the substitute explanatory concept, and the student believes that the substitute concept has a value for resolving problems, the educator can give them a chance to pursuing a new idea instructed. If not, the error remedial must restart as soon as possible.

Methodology

This study complied quantitative and qualitative descriptive research with the steps: gathering data, interpreting and analyzing data, and reporting the findings (Creswell, 2013; Nassaji, 2015). We used it because we wanted to understand students' in-deep misconceptions in defining an angle and the triangle before they attended a geometry course at the beginning of the first semester. Our investigation had the following three main objectives. (1). To identify students' common errors, the roots, and the causes of misunderstanding in defining the angle and the triangle. (2). To investigate the metacognitive regulation scheme in guiding and leading student thinking of geometry concepts. (3). To introduce the MDWI model for defragmenting, reconstructing, or linking students' knowledge from the incorrect geometry concepts to be correct concepts about angle and triangle definition.

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Participants and Times
The research involved two mathematics education department students groups with 40 students per class from an private university in Mataram, Indonesia. Both groups were undergraduate students. They graduated from public and private high schools and passed the national mathematics examinations included the geometry lesson. The research was held from March until December 2021.

Instruments
The instruments of this research used open-ended question tests about angle and triangle concepts to investigate students' errors and misconceptions of both geometry terms. The contents consisted of 3 questions associated with constructing these terms and writing into formal definitions. Each item of this test instrument was available for the students' opinion questions (Appendix). Referring to the students' test answer errors, we interviewed each student to discover their existing concept errors and misconceptions dealt with his/her formal definitions composed. Using the table, we classified the common error types, the roots, and the causes of misunderstanding (Table 1).

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Procedure

- *Identifying common errors and diagnosing the roots and the causes of misunderstanding*

Finding common errors and the roots of misunderstanding about defining angle and triangle were using open-ended questions test in Appendix A. The results of student work were evaluated and analyzed related to the occurrence of student answer errors. Then it was classified and coded from high common errors level (E1) to low errors (E5), as shown in Table 1. Discovering the roots of their misconception, ensuring learners mistake from the test answers (whether it was an error or misunderstanding), and improving awareness of students from the thinking, we interviewed them 1-1 through a personal approach related to their incorrect answers. The interview content was related to their experiences about the students' geometry pre-knowledge, ways of thinking, oral describing the definition of angle and triangle, and admitting his/her errors. The results of these activities were presented in Table 2. The learners of the understand and inadequate understanding in the category of minor misconceptions respectively treated by defragmenting and connecting the incorrect geometry concepts with the right ideas. Other, it treats using reconstruction actions as shown in Table 3.

- *Misconceptions' Correction Using MDWI and Concepts Exchange Decision*

The treatment for the minor error group's students was carried out individually following the metacognitive regulatory mechanism in Figure 1. In starting correction tasks of misconceptions, the students should find out all primitive terms and some supporting concepts used to define the angle and triangle (point, line, ray, segment, position, and direction). Using these elementary geometry objects, they should try to draw and present any angle and triangle shapes. Hereafter, they explained each construction process and wrote his/her result work in the formal definitions using their language. In this case, the intervention and instruction of the educator emphasized improving and revising the following knowledge aspects. (1) recognizing the pre-knowledge for defining the geometry concepts; (2) demonstrating the concepts' construction process with pictures; (3) presenting and writing this demonstrated concept idea; and (4) composing the formal definitions. The corrective function of the treatments is to fill the gap (hole) of students' conceptual understanding or revise and reconstruct the concepts' wrong parts.

On the other hand, the treatment for learners from the acute error group remediated the same as those in the minor misconception group. Still, we divided them into some groups (3 students per group). The group function was to discuss and evaluate the alternative definitions that resulted from the group members. Hereafter, each group should finally produce some geometry alternatives definitions that were most suitable with the formal concept discussed. During this step, the educator used some counterexamples and cognitive conflict strategies to make students dissatisfied with their ideas. These ways are also applied to straighten students' thinking, guide, and help them to exchange the students' incorrect concepts with correct concepts. The scheme and results of these treatments are introduced in Table 4.

The educator evaluated the students' difficulties correction results of both minor and major misconception levels. If it considered that both groups' works were correct, clear, and coherent with the formal concepts, then they could pursue a new idea. If not, they should repeat in-deep.

Results

Problem 1: Identifying Common Errors and Diagnosing Roots and Causes

Based on the analysis results of student answers of the test items in the Appendix, it founded that thirty-seven students made errors in defining angle, and there were sixty-one students in explaining triangle. In this case, this research identified five types of students' common errors. First, students assumed that an angle is a point (**E1**). Second, they thought that an angle is the area part of a plane between two legs of this angle (**E**2). Third, students argued that an angle is a figure represented by two line segments combined at one endpoint of both line segments (**E3**). Fourth, they stated that a triangle is a part of a plane piece that forms the triangle (**E4**), and, fifth error, they concluded that any three line segments define a triangle (**E5**). On the other sides, in answering test item 1, there were 14 students (17.5%) of the error **E1**, 11 students (13.8%) of the error **E2**, and 12 students (15%) of the error **E3**. For the test items 2 and 3, respectively, there were 28 students (33.8%) of the error **E4** and 33 students (41.3%) of the error **E5**. Thus, the total errors of the test items 1, 2, and 3 were successively 37 students (46.3%), 28 students (33.8%), and 33 students (41.3%). The misconceptions frequencies (f) of these thirty-seven learners in solving geometry test items 1, 2, and 3 see in Table 1.

Table 1. Frequencies of Students' Misconception in Understanding Angle and Triangle

Problems	**Error 1 (E_1)**		**Error 2 (E_2)**		**Error 3 (E_3)**		**Error 4 (E_4)**		**Error 5 (E_5)**		**Total**	
	f	%	f	%	f	%	f	%	f	%	f	%
Test item 1	**14**	17.5	**11**	13.8	**12**	15	-	-	-	-	**37**	46.3
Test item 2	-	-	-	-	-	-	**28**	33.8	-	-	**28**	33.8
Test item 3	-	-	-	-	-	-	-	-	**33**	41.3	**33**	41.3

The errors' roots and causes of students misconceptions have resulted from the student works analysis and in-depth interviews. The interview content was related to their experiences about geometry pre-knowledge, ways of thinking, oral describing and writing the definitions of angle and triangle, and admitting errors. From the interview results, we also evaluated the missing and incomplete knowledge structures, the connection among the learned material of the students, and the logical consequences as follows.

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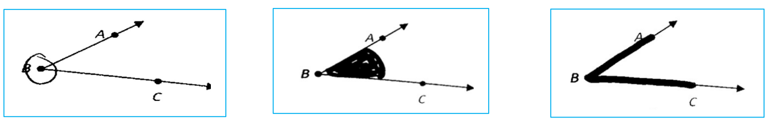
Misconception 1: Over-specializing that an angle $\angle ABC$ is the point B.
Students who undergo this misconception could memorize that an angle definition $\angle ABC$ is the union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersected at the common endpoint B, but they mark point B as an angle (Figure 3a). Regarding results of interviews with the students of the misconceptions **E**1, did not generally understand some undefined terms and geometry's elementary objects used for defining rays, line segments, and drawing an angle. They also could not explain the relationship between these concepts to construct the angle. Moreover, they ignored the starting point position and the direction for a line ray and habited the writing angle symbol with only one capital letter. Due to the learners define the angle maybe with rote, consequently, they say that this angle is the point B.

Misconception 2: Over-generalizing that an angle is the area part of a plane bounded by two legs of this angle.
This misunderstanding appeared in the students' answers to Test Item 1 about the problem of angle. The learners already known an angle constructed by two rays intersect at the endpoint, but they stated an angle figure as the area bounded by their angles legs. For example, Figure 3b shows the work of a student who has this second misconception type. The in-deep interviews could inform the students' misconceptions from two causes. They did not understand that two rays met at the starting point would consistently result in the rays pieces connection (not a cut of plane), and they lacked the prior knowledge to differentiate between

an angle and its measure. As a result, they have the misconception that an angle is the area part of a plane bounded by two legs of this angle.

Misconception 3: An angle is a figure formed by two line segments that meet at one endpoint of the segments.

In the third misconception, the students recognized that the angle $\angle ABC$ was a union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ coincided at its starting point B, but they stated that an angle $\angle ABC$ was two line segments $\overline{BA}$ and $\overline{BC}$ that met at point B. For example, Figure 3c shows the works result of students that the line segments $\overline{BA}$ and $\overline{BC}$ as an angle. They argued an angle $\angle ABC$ was a set of points of line segments $\overline{BA}$ and $\overline{BC}$. Appertaining to the results of in-depth interviews, the students who make the misconceptions have not consistently differentiated between segments and rays to define an angle. They over-specialized this angle represented with three points and three capital letters. Because of this, the students said an angle was a figure formed by two line segments that meet at one endpoint of the segments.



(a) (b) (c)

Figure 3. Student work results in the completion of the test items 1

Misconception 4: A triangle is a part of a plane piece that forms the triangle.

Students consider a triangle as the interior of the triangle or the area bounded by the triangle. This misconception appeared in the students' works of test item 2 relating to the triangle concept. In this test item 2, the students who had a misunderstanding could define a triangle as a polygon of three sides but pointed out the graph that the triangle was an area bounded by the sides of the triangle (interior of the triangle). For example, Figure 4a shows the works of students experiencing the fourth misconception. Referring to the results of interviews, the students error **E**4 did not know that the merging three line segments at its endpoints for constructing triangle would produce three line-segments connection picture. They also used their primary school experiences in which a triangle was made from cutting paper through three noncollinear points. In consequence, these learners declared a triangle is a part of a plane piece that forms the triangle.

Misconception 5: Any three line segments define a triangle.

Students concluded that any three line segments can form a triangle. This misconception happened from test item 3 about three line segments as data for triangle sides. Learners understood that triangle sides had three line segments. Relating to the solution of test item 3, Figure 4b, the students explained that any three line segments could form a triangle, i.e., a right triangle or other triangles. Because of these triangle images and without examining the measure of these three line segments data, they made wrong conclusions.

(a) (b)

Figure 4. Students Work Results in the Completion of the Test Items 2 and 3

Appertaining to these students results of interview, the errors **E**1 to **E**5 found ten information about the roots of students' misconceptions. The list of these roots and causes of students' misunderstanding presents in Table 2.

~~*Tabel*~~*Table 2. Common Errors, the Roots and the Causes of Students' Misunderstanding*

Code	Common errors	The Roots and the Causes of Misunderstanding (Code)
E_1	An angle $\angle ABC$ is a point *B*.	1. The students do not understand some undefine terms and geometry's elementary objects used for defining rays, line segments, and drawing an angle (**E_{11}**). 2. They do not know the relation between these definitions to construct an angle (**E_{12}**). 3. They define the angle with the rote and habit the writing angle symbol with only one capital letter (**E_{13}**).
E_2	An angle is an area part of a plane bounded by two legs of the angle.	1. The learners do not understand that if two line rays meet at their starting point will consistently result in the rays' pieces connection (**E_{21}**). 2. They lack the prior knowledge to differentiate between an angle and its measure or the area between their angle legs (**E_{22}**).
E_3	An angle is a figure formed by two line segments that meet at one endpoint of the segments.	1. The learners can not consistently differentiate between segment and ray to define an angle (**E_{31}**). 2. They over-specialize an angle represented with three points and three capital letters (**E_{32}**).
E_4	A triangle is a part of a plane piece that forms the triangle.	1. The students do not know the merging three line segments at their endpoints for constructing a triangle will produce a line-segments connection picture (**E_{41}**). 2. They have a misconception from primary school experiences in which a triangle make from cutting paper through three noncollinear points (**E_{42}**).
E_5	Any three line segments define a triangle.	1. The students state the conclusion for defining a triangle using some triangle images without counting and comparing the length of three line segments (**E_{51}**).

Problem 2: Misconceptions' Correction Using MDWI and Concepts Exchange Decision

In general, we found the sources of these misconceptions of students were the lack of prior knowledge or missing knowledge of geometry concepts (MK), the existence of knowledge gap or incomplete knowledge (IK), interpretation deviation of concept images (ID), feeble logical thinking (FT), and low connection of students knowledge (LC). These causes characterize the misconceptions in the following three types. In case the causes MK and IK, we call inadequate understanding with minor errors (10 students). For the causes ID and FT, we state inadequate understanding with major mistakes (18 students), another (LC), it calls almost understanding with minor errors (9 students). Resolving these misconceptions' causes, we introduce the approach to reconstruct thinking fragmentations of MK and IK, rearrange knowledge structures and logical thinking of ID and FT, and link knowledge of students LC. For this solutions approach, we state, respectively, with the terms: defragmentation, reconstruction, and connection solutions as shown in Table 3.

~~*Tabel*~~*Table 3: Treatment Approaches of Students' Misunderstanding*

Concepts Mastery Achievement	**Number and kinds of Students Misconception and Treatment Types**	
	Minor	**Major**
Inadequate understanding	10 Students of MK and IK *Defragmentation*	18 Students of ID and FT *Reconstruction*
Almost understanding	9 Students of LC *Connection*	-

This section reports the errors correction of students using the MDWI approach. It cures the students' misunderstanding roots in Table 2 for their error cases in Table 1. The stages of the MDWI were as follows (Table 4). Implementing the metacognitive regulation scheme presented in Figure 1, the instructor directed the students to learn primitive concepts of the angle or triangle (activity M1) and design figures of the angle or triangle (activity M2). Evaluating these students' activities was focused on drawing an angle or a triangle idea connected with primitive concepts. The stage of activities M1 and M2 is called the drawing concept (D). Then, the students presented pictures and wrote geometry ideas based on prior knowledge (activity M3) and composed formal definitions (activity M4). The instructor helped the learners to recognize the geometry characteristics and connections of the angle and triangle elements, the logical thinking for constructing an angle or triangle and write the definitions of angle and triangle through the pictures. The stage of activities M3 and M4 is called the writing task (W). During the learning activities D and W, the instructor gave interventions and instructions to develop the learners' knowledge structure and induce cognitive conflicts. These cognitive conflicts are designed to resolve the causes of students' misunderstanding MK, IK, ID, FT, and LC. The intervention actions of the cognitive conflict, i.e., respectively, marked by the code $\mathbf{I}_{MK}$, $\mathbf{I}_{IK}$, $\mathbf{I}_{ID}$, $\mathbf{I}_{FT}$, and $\mathbf{I}_{LC}$ presented in columns 4-7 in Table 4. Students with minor misconceptions were individually treated, in contrast, to the major misconceptions, remedied in group of five students. Assessing students' achievement with scores interval 0–100 and in-depth interview, the treatments found the average result scores shown in column nine of Table 4.

Some cognitive conflicts examples associated with the treatments of students' misunderstanding MK, IK, ID, FT, LC in Table 2 and often used by the instructor for interventing students in this research were as follows.

1. An angle $\angle ABC$ is just a point B that is undefined geometry object; versus a set of points consisting of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersects at the endpoint *B*.
2. An angle $\angle ABC$ is a measure of arc degree or an area between two angle legs $\overrightarrow{BA}$ and $\overrightarrow{BC}$; contra to the joint of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ coincides at the point *B*.
3. An angle $\angle ABC$ is a joint of two line segments $\overline{BA}$ and $\overline{BC}$ meet at the endpoint *B*; against the union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersects at the endpoint *B*.
4. An angle $\angle ABC$ is just three points *A*, *B*, and *C*; versus a union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ meets at the point *B*.
5. The joining of three line segments $\overline{BA}$, $\overline{BC}$, and $\overline{AC}$ at their endpoints *A*, *B,* and *C* will form a cut of a plane *ABC* called a triangle *ABC*; contra to they produce a picture of three-line segments called a triangle *ABC*.

6. Every three line segments $\overline{BA}$, $\overline{BC}$ and $\overline{AC}$ can form a triangle *ABC*; contra to the line segments of the measures $\overline{BA}$ = 3 cm, $\overline{BC}$ = 5 cm, and $\overline{AC}$ = 15 cm will not construct a triangle *ABC*.

~~Tabel~~Table 4: Errors Treatment Using MDWI

Errors Roots	**Students Number**	**Treatment Types**	**Metacognitive Regulation (M)**				**Correct Students Number**	**Average Result Scores (0 – 100)**
			Drawing Concept (D)		**Writing Task (W)**			
			M1	M2	M3	M4		
E_{11}	**7**	Defragmentation, Reconstruction, & connection.	**I_{MK}**	**I_{IK}**	**I_{ID}**	**I_{FT}-I_{LC}**	**5**	**71.43**
E_{12}	**4**	Reconstruction.	-	-	**I_{ID}-I_{FT}**	**I_{FT}**	3	**75.00**
E_{13}	**3**	Reconstruction & connection.	-	-	**I_{ID}**	**I_{FT}-I_{LC}**	2	**66.67**
E_{21}	**6**	Reconstruction	-	-	**I_{ID}**	-	4	**66.67**
E_{22}	**5**	Defragmentation & reconstruction.	**I_{MK}-I_{IK}**	**I_{ID}**	**I_{ID}**	-	4	**80.00**
E_{31}	**7**	Defragmentation & reconstruction.	**I_{MK}**	**I_{ID}**	**I_{ID}**	-	5	**71.43**
E_{32}	**5**	Reconstruction.	**-**	**-**	**I_{ID}**	**I_{FT}**	4	**80.00**
E_{41}	**17**	Reconstruction & connection.	**-**	**-**	**I_{ID}-I_{LC}**	-	13	**76.47**
E_{42}	**11**	Reconstruction.	**-**	**-**	**I_{ID}**	**-**	8	**72.73**
E_{51}	**33**	Reconstruction & connection.	**-**	**-**	**I_{ID}**	**I_{LC}**	24	**72.73**
Total	**98**	**Number of Interventions**	**4**	**3**	**12**	**7**	**72**	**73.31**

Discussion

Based on Table 1, it informs that only 43 students (53.7%) are correct in understanding the concept, and 37 students (46.3%) can not define the angle properly. On the other hand, it finds 33 of the 61 students (76.3%) who fail in understanding the triangle idea. Then, only 19 students (23.7%) are successful in clarifying the concepts. Thus, there are no more than 54% of the students can correctly define both angle and triangle terms.

Evaluating the roots and the causes of misunderstanding in Table 2, these can identify the impediments and feebleness of students. The students do not know the primitive terms, the function, and the role of the ray and line segment in defining an angle or triangle. In this case, they learn the geometry concepts and definitions partially. These results were relevant to the studies of Ozkan et al. (2018), Al-Khateeb (2016), and Özerem (2012), who informed that one of the misconceptions causes was the lack of prior knowledge and insufficient students' knowledge of the geometry concept. In addition, it made misconceptions in reasoning and basic operation mistakes. Besides, the mathematical material was, generally, interconnected. So, this study should eliminate the previous errors before introducing a new issue. After that, the misconceptions may happens to some images interpretation errors in the geometry concepts understanding. For example, the students undergo the visual deviation between a point and an angle, an angle measure and an angle, and between a triangle and a plane cut triangle. Then, they have trouble reasoning for constructing an angle and triangle using the rays and line segments. There is limited understanding of geometry relations between the points, lines, rays, and line segments to determine an angle and triangle. These finding results is in line with the studies of Poon and Leun (2016), Biber et al. (2013), Cunningham and Roberts (2010), and Gal and Linchevski (2010) who found that learners faced difficulties in selecting the characteristics of figural elements relevant to the concepts. Due to the students focused only on the physical shapes and the geometry images rather than identifying the essential geometry properties of represented figures and fundamental logical reasoning abilities.

Handling the students' misconceptions as presented in Table 4 shows that it is necessary for at least 26 interventions provided by the instructor. Implementing these strategies was based on the metacognitive regulation scheme (M) that focus on solutions M3 (12 interventions), M4 (7 interventions), M1 (4 interventions), and M2 (3 interventions). The treatment actions resolve the students' difficulties with the reconstruction approach at least seventeen times, the connection way four times, and the defragmentation technique five times.

Referring to the frequencies amount of the interventions $\mathbf{I}_{\mathrm{ID}}$ (12 times) and $\mathbf{I}_{\mathrm{FT}}$ (5 times), it can conclude that, in general, students' misconceptions are the interpretation deviation of the image concept to define angle or triangle. The source of this main problem is that the students fail to recognize the geometry objects for drawing the angle or triangle (activity M2) and flunk to clarify, logically of how to construct this angle or triangle by using these objects (activity M3). Because this activity is not passed well by students, it is naturally misperceive that an angle is a point, three points, or a combination of line segments. Furthermore, they cannot distinguish that the triangle is a combination of three line segments, but they perceive it as the interior area of the triangle. The students make a wrong generalization (over-specializing and over-generalizing). Then, the frequencies $\mathbf{I}_{\mathrm{MK}}$ (3 times) and $\mathbf{I}_{\mathrm{IK}}$ (2 times) indicate that, for activity M1, the students do not have much knowledge about the geometry elementary objects (primitive terms) and definitions to construct the angle and triangle. Hence, students assume that an angle represented with the legs of line segments or rays pictures is equal. They also consider the angle as three points (concept images errors). On the other hand, the frequencies $\mathbf{I}_{\mathrm{LC}}$ (4 times) signify that, for activity M4, the learners are doing errors to describe and define an angle and triangle concepts with their words, due to, they fail to connect the features of points, line segments, and rays to construct the angle and triangle. As a result, they define the angle with rote or using some triangle images.

From Table 4, we can review the MDWI method for correcting the student misconceptions as follows. The defragmentation, reconstruction and connection treatments for the error root $\mathbf{E}_{11}$ using interventions $\mathbf{I}_{\mathrm{MK}}$, $\mathbf{I}_{\mathrm{IK}}$, $\mathbf{I}_{\mathrm{ID}}$, $\mathbf{I}_{\mathrm{LT}}$ and $\mathbf{I}_{\mathrm{FC}}$ in metacognitive regulation activities M1, M2, M3, and M4 give the score of 71 (very good). Moreover, the defragmentation and reconstruction treatments $\mathbf{E}_{22}$, and $\mathbf{E}_{31}$ with interventions $\mathbf{I}_{\mathrm{MK}}$, $\mathbf{I}_{\mathrm{IK}}$, and $\mathbf{I}_{\mathrm{ID}}$ in metacognitive regulations M1, M2, and M3 remedied the students in identifying the angle fitures with the average correct result score of 80 (excellent). In case the interventions $\mathbf{I}_{\mathrm{MK}}$ and $\mathbf{I}_{\mathrm{ID}}$, they achieve the average score of 71 (very good). The overall reconstruction and defragmentation treatments could avoid significantly the incomplete and missing knowledge of students, and the interpretation errors of the angle concept images.

The reconstruction treatments $\mathbf{E}_{12}$, $\mathbf{E}_{21}$, $\mathbf{E}_{32}$, $\mathbf{E}_{42}$ with intervention I_{ID} in activities metacognitive regulation M2 and M3 declined the student errors in drawing, recognizing features, and explaining ideas in the constructing angle and triangle. The students could attaint the average correct result score of 70 (very good). Besides, in the interventions $\mathbf{I}_{\mathrm{ID}}$ and $\mathbf{I}_{\mathrm{FT}}$, they achieved the average score of 78 (very good). These reconstruction treatments, in general, improved their skills in demonstrating figures, describing the ideas, and making generalizations to define the angle and the triangle.

The reconstruction and connection treatments $\mathbf{E_{13}}$, $\mathbf{E_{41}}$, and $\mathbf{E_{51}}$ with interventions I_{ID} and I_{LC} in metacognitive regulation activities M3 and M4 remedied the learners' errors of the concept images and the interconnecting of geometry concepts. The students could achive the average correct result score of 75 (very good) in the angle and triangle comprehension. For the interventions $\mathbf{I_{ID}}$, $\mathbf{I_{FT}}$ and I_{LC}, it finds the score of 67 (good). These treatments could increase the students skills in reconstructing and connecting previous geometry concepts to define the angle and triangle.

Based on the remedies results score of 73 (very good), and the students of the correct answers of 72/98 = 73 %, it could state that the overall MDWI treatment approach could help the students to exchange their misconceptions in defining angle and triangle from misconception concepts to be correct concepts. However, it suggested that, in constructing the definitions of geometry ideas, the students have to custumize using this metacognitive regulation scheme. If not, otherwise it is worried that they will only rote these concepts. These students' corrections result in averages come across that their works have been correct and coherent with the formal definitions, and each student's score was more than 60. Thus, they could pursue a new learning topic. To sum up, these interventions scheme and MDWI method resolve the student errors in generalization, concept images, and connection incompetencies of geometry features that were discussed by Gutiérrez and Jaime (1999), Özerem (2012), Poon and Leun (2016), Ozkan et al. (2018), and Şahin et al. (2020).

Conclusion

The common errors occurred in defining angle and triangle of mathematics education students, i.e., the assumption and generalization that angle is a point, an area part of a plane between two legs of the angle, and a union of two line segments that meet at one endpoint of the segments. Then, they also made the misconceptions that a triangle is a part of the plane piece forms the triangle, and any three line segments define a triangle. The roots of these errors, generally, are of basic ideas. They do not know the primitive terms, the function, and the role of the ray and the line segment for defining an angle or triangle. Moreover. their understanding of geometry concepts and definitions is still partially. The students commonly fail in concept images interpretation, logical thinking, and knowledge connection needed to draw, construct, and write the definitions of angle and triangle.

Error correction using the MDWI treatment approach guides the students to learn primitive concepts, draw the geometry idea connected with primitive ideas, present and write these pictures into formal definitions. During this learning-teaching process, the instructor gives interventions and instructions to develop their knowledge structure, induce cognitive conflicts, and resolve the causes of their misunderstanding relating to the generalization errors, interpretation deviation of concept images, and connection incompetencies of geometry features. In most cases, these treatments effects empowered the students to decline and reduce their errors in recognizing the geometry objects and generalization, interpretation concept images, and connection of geometry features.

Recommendations

Considering error correction is rare in defining geometry objects, this MDWI treatment provides a guide for cure the learners of the geometry concepts misconception step by step based on the metacognitive regulation. In line with the university learners' errors amount of these generalizations and concept images, the treatments M2 and M3 are very effective for helping them to avoid the misunderstanding of interpretation deviation and logical thinking of concept images and geometry features. Based on this research results, it is suggested that future research is tried the junior and high school students. It is necessary because they have to learn a lot of primitives and definitions of geometry's elementary objects and comprehend some fundamental theorems. We can suspect that some students will be misperceptions in understanding the concepts and ideas. They may also meet difficulties in logical reasoning to explain or prove these theorems.

Limitations

This research was conducted on students elected from an private university in NTB Province, Indonesia. Thus, the generalization of the results in this research has limitations. Another limitation of the study was that it only focused on defining the terms of geometry concepts.

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Appendix

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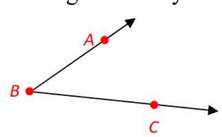
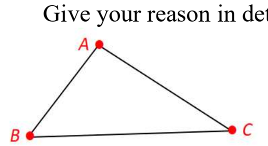
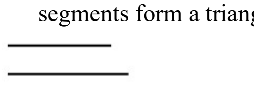
Instruments for Identifying Angle and Triangle Misconceptions and Types of Problems

Problem	**Problems Types**
1. Consider the points *A*, *B*, and *C* in Figure 1. Give a mark using a colored pen, which is a part of Figure 1 called an angle? Give your reason in detail!  **Figure 1.** Angle	Angle Problem
2. Let three points *A*, *B*, and *C* in Figure 2. Give a mark using a colored pen, which is a part of Figure 2 called a triangle? Give your reason in detail!  **Figure 2.** Triangle	Triangle Problem
3. Given any three line segments in Figure 3. Can these line segments form a triangle and explain why!  **Figure 3.** Three line Segments	Line Segments Problem on Triangle

BUKTI CORESPONDING

Identifying and Correcting Students' Misconceptions in Defining Angle and Triangle

European Journal of Educational Research

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Manuscript ID:	EU-JER_ID# 22012011532730	Date:	April 23, 2022
Manuscript Title:	Misconceptions Correction of Mathematics Education Department Students: A Case Study in Defining Angle and Triangle		

ABOUT MANUSCRIPT (Mark with "X" one of the options)	Accept	Weak	Refuse	Not Available
Language is clear and correct		X		
Literature is well written	X			
References are cited as directed by APA		X		
The research topic is significant to the field	X			
The article is complete, well organized and clearly written	X			
Research design and method is appropriate	X			
Analyses are appropriate to the research question	X			
Results are clearly presented	X			
A reasonable discussion of the results is presented		X		
Conclusions are clearly stated	X			
Recommendations are clearly stated		X		

GENERAL REMARKS AND RECOMMENDATIONS TO THE AUTHOR

Although overall acceptable, grammar and language of the manuscript need a revision.

Please, fix style of the manuscript. Use APA 7th ed. manual and journal's template.

All figure and table titles should be in title case.

A longer recommendation for future research is needed in Recommendations section.

THE DECISION (Mark with "X" one of the options)	
Accepted: Correction not required	
Accepted: Minor correction required	X
Conditionally Accepted: Major Correction Required (Need second review after corrections)	
Refused	

Reviewer Code: R2613 (The name of referee is hidden because of blind review)

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Language is clear and correct		X		
Literature is well written		X		
References are cited as directed by APA	X			
The research topic is significant to the field		X		
The article is complete, well organized and clearly written		X		
Research design and method is appropriate		X		
Analyses are appropriate to the research question	X			
Results are clearly presented	X			
A reasonable discussion of the results is presented		X		
Conclusions are clearly stated		X		
Recommendations are clearly stated		X		

GENERAL REMARKS AND RECOMMENDATIONS TO THE AUTHOR

Introduction

My major concern with the introduction is that rationality is not explained based on the literature. The authors give some information about the problem in Indonesia. However, they do not present no information about the existing literature for an international audience. It should be noted that readers of this international journal want to prefer to read about the problem in the international literature. From this perspective, the authors need to emphasize the research gaps in the literature. Mainly, there is a need to emphasize the weakness and strengths of the existing studies on geometry, misconceptions, and the MDWI.

What is the MDWI? I could not find its full name in the text.

The authors wrote that "Several result studies informed that students failed to complete dimensional deconstruction of images to obtain mathematical properties". So what are several studies? And what are the main results of these studies?

Literature Review

The literature review is like an introduction. Less number of studies have been presented some findings. However, the review does not present the main results of previous studies about geometry, misconceptions, and the MDWI. In this section, after presenting the main results of the research studies, there is a need to make an overview of the existing literature.

Problems and Purposes of Research

Because the need for the current study is not explained well, I think the research questions do not stem from the literature. For instance, what are the results from studies about the angles and the triangles? What do we know about geometry, misconceptions, and the MDWI? What do not we know about these dependent variables?

Framework of Research

The writing in this section is too general and also overly long. The information given is somewhat confusing. I

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think the focus of the study is missed in this section. If the aim is to explain the implementation of the MDWI approach, then the information given in this section should be moved to the method. To be honest, I could not understand what the authors sought to mention.

Method
Delete the word “excellent”. What are the criteria for using this word? What is the authors’ reference?

How the instrument was developed? How the validity and reliability were ensured?

How the data were analyzed? There are no details about this detail.

My major concern is that it is not clear how the MDWI was implemented during the data collection?

Discussion
The discussion is a repetition of the results. The authors should focus on differences and similarities between previous studies and this study. They should also discuss the differences and similarities in this section. In addition, possible reasons for the results should be discussed.

Conclusion
What is the contribution of this study to the literature? What is the new knowledge for the literature? Please clarify these details.

Recommendations
Implications for further studies are missing. Please explain what should be researched by scholars in the future?

Language
The language is not acceptable. I had many difficulties while reading the text. A native speaker should edit the paper.

Overall, I think major revisions are needed to increase the quality of this paper.

THE DECISION (Mark with "X" one of the options)	
Accepted: Correction not required	
Accepted: Minor correction required	
Conditionally Accepted: Major Correction Required **(Need second review after corrections)**	X
Refused	
Reviewer Code: R2612 (The name of referee is hidden because of blind review)	

Misconceptions Correction of Mathematics Education Department Students: A Case Study in Defining Angle and Triangle

Abstract: Misconceptions are one of the most handicaps to comprehend in learning mathematics. This study aimed to investigate the common errors and the misconception causes of students in defining the angle and triangle. Moreover, the research introduces the MDWI treatment approach to exchange the understanding from the wrong concepts to be the correct concepts of these geometry ideas. To achieve this goal, it used a research design. It identified and resolved the errors in defining the angle and triangle of the first-year undergraduate students of the mathematics education department from an excellent private university in Mataram, Indonesia. The steps were as follows. Using open-ended questions tests and in-depth interviews, we identified the errors, the roots, and the causes of learners' misunderstanding. Then, implementing the MDWI treatment introduced the way to correct these errors correction. The research found that learners generally fail in concept images interpretation, logical thinking, and knowledge connection needed to define the angle and triangle. The MDWI approach remedied the misconceptions of generalization, concept images errors, and connection incompetencies of geometry features.

Keywords: *Misconception correction, common errors, cause, treatment, angle and triangle.*

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Introduction

It reported the ability of school students in Indonesia is still low. The results of tests and evaluations from the International Student Assessment (PISA) along the years 2015 and 2016 undertaken by the Organization for Economic Cooperation and Development are classified as low completeness (OECD, 2016). The tested material also includes geometry. It means that students have not fully mastered geometry's main subjects, including quantity, relationship, and uncertainty (NCES, 2004). In 2019, the mathematics score's national exam of junior high school students failed, namely in the position 45 of score interval 0-100. Besides, for senior high school level, the student's achievement to answer all given questions of the geometry and trigonometry test with correct responses category was only 37% in 2017; and 34% in 2018. Mastering the learners' mathematics was still not good, i.e., in average score 45 of interval 0-100 (Kemendikbud, 2019). These fail cases will potentially be repeated by the school students in future studies, particularly in the first year of an undergraduate program.

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The learners' weak understanding of geometry definitions and concepts as the geometry pre-knowledge will affect low future mastering geometry, difficulties, and failure. One factor in which the students can happen is the geometry misconception when their concentration only focuses on the physical shape and the geometry images rather than identifying the essential geometry properties of represented figures (Poon & Leun, 2016; Biber et al., 2013). Based on the geometry questions to the students about the definitions of angle, measure, and shape, this study reported a lack of students' background knowledge that makes many learner misconceptions in reasoning and basic operation mistakes (Özerem, 2012). These facts indicate that the learners have to develop their understanding of geometry concepts and some related abilities.

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A theoretical and passive teaching approach, be likely to memorize, and the few visuals provided do not guarantee that students can master the definitions and geometry concepts. Stimulating and challenging geometry concepts is required that students will be able to improve their understanding. Several result studies informed that students failed to complete dimensional deconstruction of images to obtain mathematical properties. They found it difficult to determine the characteristics of figural elements relevant to the concept (Cunningham & Roberts, 2010; Gal & Linchevski, 2010). This misunderstanding can occur because of the teachers themselves or their environment, i.e., the proficiency and inadequate book facilities. Although the teachers' role in developing knowledge is essential, they also need to play an active role in dealing with their misunderstanding problems. Through metacognitive activities, such as their own written work error analysis, students can find and try to align their conceptual inconsistencies with more formally accepted mathematical constructs (Tirosh 1990 in Kembitzky, 2009). This conceptual change needs some strategies and metacognitive skills. Referring to Stepans' model of conceptual change, to change the alternative concepts of students needs the nature of learning tasks that can help learners exchange their understandings with the right ideas. The nature of the learning environment can involve social dialogs and negotiations among learners (Sarar, 2014). Because of these geometry learning impediments, we have to make aware and strengthen the conceptual understanding in the error correction of geometry lessons. Consequently, it needs a new learning treatment approach to improve the correct geometry concepts and reasoning. For this reason, using metacognition regulation (M), drawing geometry concepts (D), students' writing tasks (W), and lecturers' intervention effort with the social involvement approach (I), we will evaluate the students' error correction results in the teaching-learning process. In short, this research will apply the MDWI teaching-learning method for exchanging the students' concepts errors to be the correct concepts.

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Literature Review

Metacognition is vital in supporting the performance of cognitive tasks in mathematics learning. Some research results reported that metacognitive understanding includes aspects of cognition's knowledge and cognition's regulation. This knowledge contains the cognitive abilities, processes, resources, and the influence of a person, task, or strategy factors on performance (Brown & Palincsar, Flavell & Wellman in Garofalo et al., 1985). The regulation of metacognition is concerned with the strategic decisions' activities in a course via cognitive tasks. These activities consist of planning studies, monitoring processes, evaluating and revising the outcomes. Robert J. Sternberg (in Artzt & Thomas, 2002) stated that metacognition is various. It includes both understanding and control of cognitive processes that include planning, monitoring, and evaluating activities. This understanding process, of course, must be effective action. It also must be remembered that metacognition interacts with many other aspects of the student, i.e., abilities, personality, and learning styles. Magiera & Zawojewski (2011) used metacognitive awareness, regulatory, and evaluative approaches for small-group learners. They identified and characterized the social-based and self-based contexts related to their metacognitive activities in mathematical modeling learning. Based on these research studies, the metacognition approach supports the learning process in mastering concepts and geometric reasoning. The application of metacognition regulation expects to prevent students from thinking that they only memorize concepts. It can be a tool to make learners aware and correct their reasoning weaknesses in mastering the concept of geometry. In the learning process, the teacher can use it to determine the weak points of student reasoning, develop the strategies for learning, and determine some stages of student error correction.

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Visual geometry objects help build people's experiences, beliefs, and understanding of an item through a cognitive process. Logically, the images of geometry objects are useable to explain the relationship between one and another concept. The efficacy of visualization will help understand concepts and support learners to acquire educationally required knowledge (Phillips et al., 2010). On the other sides, as a tool in learning geometry, images of geometry objects are widely used to explain definitions and concepts of geometry, but some learners still have misconceptions. Students usually know the ideas from their memory associated with the figures. Unfortunately, they fail to match both concept's formal definition and the geometrical figures (Vinner & Hershkowitz, 1980). Berthelot & Copy (in Poon & Leun, 2016) stated that one of the students' misconceptions factors in geometry learning is an incapability to identify various shapes (symbolic, visual, etc.) of the same geometry concept. Özerem (2012) found the student feebleness of measures, angles and shapes, transformations and construction, and 3-D shapes.

Many misconceptions students also reported the lacking of background knowledge, reasoning, and basic operation mistakes. Hence, applying geometry images to aid the learners in understanding abstract geometry ideas, the teachers need to identify, verify, and evaluate the understanding errors and misconceptions of ideas. Harmonizing the concept images seen and what students have in mind can use some geometry pictures to develop cognitive conflict strategies and counterexamples for the learners. In learning, understanding a geometry concept of the students can have misconceptions, namely different from her/his accepted ideas. It is a real challenge for university educators in which they generally refuse to be corrected. Therefore, providing personalized interventions to help learners resolve misunderstandings in this context is a difficult challenge. Educators must work with their learners to identify, recognize, and correct commonly held misconceptions to attain the best learning outcomes. Any student misconceptions critically need to be evaluated, revised, and changed with information consistent with the accepted concepts (A Handbook for Educators, 2016). Generally, there were five causes of errors: language misconceptions, spatial information difficulties, deficient mastery of prerequisite skills, facts, and concepts; fallacies of thinking; and the application of irrelevant rules or strategies (Radatz, 1979 in Kim, 2011). Ay (2017) reviewed the errors. It states that apart from the test, one of the most appropriate ways for detecting these students' misconceptions is collecting qualitative data through interviews or observations. This data can provide in-depth information about students' knowledge. However, the researchers did not prefer going on further steps. Only a few studies applied a treatment to see whether the therapy could eliminate students' present misconceptions.

Teacher's teaching and intervention strategy can make a difference in students' comprehension, which is essential in instructional practice and student learning. In geometry learning, Lim state that the information's communication at the different level of reasoning among the teacher and student becomes a common cause of misconception. When teachers explain different geometry thinking levels to learners, the concepts are not fully understood or acquired. Teachers must know their students' level of geometrical understanding (in Luneta, 2015). Battista (1991) reported that developing the students' meaningful comprehension of geometry concepts requires an appropriate instructional task and assessment in teaching and learning geometry. Clarke et al. found that writing allows a teacher to see the kind of thinking and understanding that is not easy and accessible via the computational and proficiency test. Teachers can examine the process of sense-making when students explore and work with mathematics (in Kembitzky, 2009). Therefore, teacher intervention and student writing assignments will direct the achievement of conceptual understanding following curriculum objectives. It can help the learners to use previous experiences correctly and providing a new

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comprehension of the shortcomings of prerequisite material that students do not yet have and avoiding understanding concepts via rote, but by understanding processes.

The students who recognize misconceptions of geometry concepts will open their awareness of striving to improve themselves and encourages mathematics educators to develop effective strategies in helping the learners to correct the wrong ideas in the right way. However, the studies for correcting these misunderstandings in defining geometry concepts are rarely resolved. For this reason, the discussion in this paper focuses on finding common mistakes of mathematics education students about concepts of angles and triangles. Then, applying the MDWI approach would undertake misconceptions correction of the learners, i.e., using a metacognition regulation (M) and students' actions to draw all figural concepts (D) to identify the pictures' figural elements relevant to the angle and triangle concept. Then, they write the ideas (W) in definition form aided by the intervention and instruction of the lecturer (I).

Problems and Purposes of Research

Students' ability to define geometry concepts is one of the main goals of achieving the first-year undergraduate program's mathematics competencies. Unfortunately, using geometry figures and tools to support the learners in understanding the concepts and the definitions invoke some errors and misconceptions. They are difficult to determine the characteristics of figural elements relevant to the idea and often fail to arrange the words to construct the alternative definitions. To exchange the learners' incorrect concepts with the right ideas, of course, needs the nature and suitable learning methods. This research addressed to answer the following problems.

1. What do the common errors and the roots of misconceptions often happen in defining the angles and the triangles of mathematics education students of the undergraduate program?
2. How can MDWI resolve and exchange the students' misconceptions about definitions of angle and triangle from misconception concepts to be correct concepts in the teaching-learning process?

This research aimed to investigate the students' common errors and the causes of misconceptions in defining the angle and the triangle. Moreover, it introduced the benefits of MDMA for exchange the students' understanding from the wrong concepts to be the correct concepts about angle and triangle.

Framework of Research

Stage 1: Identification of Misconception

Students' misconceptions of geometry concepts can occur due to many factors such as student experiences and learning approach, teacher roles, and facilities (Poon & Leun, 2016; Özerem, 2012; Cunningham & Roberts, 2010; Gal & Linchevski, 2010). In this case, if the student's previous knowledge is likely to be lacking or there is a conflict with something learned, then the student's misconceptions need to be replaced. On the other hand, students' previous knowledge of a concept is correct, whereas understanding a new idea is not perfect, necessary to fill the understanding gap with authentic experience. Hence, determining the strategy and treating process relevant to students' misconceptions needs to identify the students' misconceptions types. From the introduced researches, it was founded some student misconceptions in geometry learning. For example, the student doesn't know that the rotation aspects contain: the direction, preserving parallel, and lengths. Some learners have errors relating to a deficiency of understanding geometry figures and insufficient knowledge of proof's importance. They also fail to state the particular polygons and their features of these polygons (Herholdt & Sapire, 2014; Cirillo & Hummer, 2019; Alamian et al., 2020; Junus, 2018).

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Stage 2: Diagnosis to Find the Roots and the Causes of Problems

The purpose of students' misconceptions diagnostic is to identify their error levels and find out the causes and the roots of their difficulties in teaching-learning geometry. Moreover, it uses to investigate the weaknesses of the teaching approach and the facilities used by educators. Using the interview method for diagnosing the errors can provide recent information of student weaknesses and flexibility of examining; meanwhile, open-ended test methods will support the students' chance to write their answers in their own words. They will probably give some new valuable responses (Gurel & Eryılmaz, 2015). This research categorized the misconceptions diagnostic of students into the following levels: almost understanding, inadequate understanding with minor and major misconceptions. Some research studies reported the concepts misunderstanding's causes that should erase. Because mathematical materials are generally interconnected, the students' misconceptions in previously discussed topics should eliminate before introducing a new issue (Ozkan et al., 2018). Al-Khateeb (2016) also states that one of the misconceptions causes is the lack of prior knowledge and insufficient students' knowledge of the geometry concept. Regarding these studies, evaluating the students' geometry pre-knowledge and inter-connecting their geometry concepts and ideas is essential to find their trouble roots.

Stage 3: Strategy and Correction of Misconceptions Using MDWI

The strategy and errors corrections use three treatment steps, i.e., student awareness; defragmentation, reconstruction, and geometry concepts connection of learners' knowledge; revision and decided to exchange from the wrong to the right ideas. The ways are as follows.

Step 1: Student Awareness of Errors and Difficulties

The first step to change the misconception is to make the student aware that there is an error from the beginning. Removing the students' impediments must crucially come from themselves, including the held beliefs and prior knowledge (A Handbook for Educators, 2016). Kruger and Dunning (2009) also warned that learners who do not know their abilities would suffer a double burden; first, they only reach wrong conclusions; second, they are problematic to raise metacognitive skills to realize it. A study by Taylor and Kowalski (2004) informed that the power of belief is a significant transitional variable that may turn over in one's mind the change process. Furthermore, Lyddy and Lambe (2013) concluded that it is easier for learners to disregard, reinterpret, or refuse new information rather than to change their beliefs. Conceptual change needs the conditions that students must be motivated, i.e., unsatisfied with their previous view. The alternative explanatory concept must be clear, i.e., helpful and coherent. Then, this substitute concept must be reasonable, i.e., rational to the learners. Finally, the substitute idea must be valuable, i.e., helps the learners in resolving problems.

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Step 2: Defragmentation, Reconstruction, and Connection Treatments of Incorrect Concept

- *Metacognitive Regulation (M)*

Supporting and guiding student errors correction is needed via the learning process. Applying the MDWI with the metacognitive processes approach: planning strategies, monitoring, and evaluation (Garofalo et al., 1985; Artzt & Thomas, 2002) will expect to resolve the students' wrong concepts through defragmentation or substitution, reconstruction, and connection processes of thinking structures. These processes are, respectively, to reconstruct thinking fragmentations of misconception, link knowledge and rectify an idea error minor, rearrange their knowledge structures and the logical thinking error. In this paper, we condition the students to be aware of the troubles as they know them from the first place, and then, the educator brings the learners to be active thinking to exchange the wrong concept with the

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right ideas. The metacognitive regulation guides the learners and instructors to design, control, evaluate, correct the understanding of geometry al features and structure the words for defining angles and triangles.

For learner errors treatment, the scheme of metacognitive regulation involves the students' understanding of the geometry pre-knowledge for constructing an angle and triangle, i.e., point, line, position, and direction. Using undefine terms, the learners must recognize the formal definitions of a line segment and a ray. Drawing, connecting, or combining among the points, the lines, the segments, and the rays, they have to try to find the shapes of the angle and the triangle. Using these constructed pictures, the students practice to explain and compose the definitions with their words. In short (Figure 1), they must resolve four treatment stages in the process of correcting misconceptions, i.e., recognizing primitive concepts and adding their pre-knowledge related to the defined geometry concepts (M1); drawing and demonstrating the concepts' figures (M2); presenting and writing geometry ideas connected with the prior knowledge (M3); composing the formal definitions (M4). These treatment series help the learners to identify, reflect, evaluate, and correct the misconceptions of angle definition and triangle in natural ways. From the teacher's side, it can quickly detect and correct the students' errors from these stages. Thus teachers' instruction and intervention for student error correction will be more focused and effective.

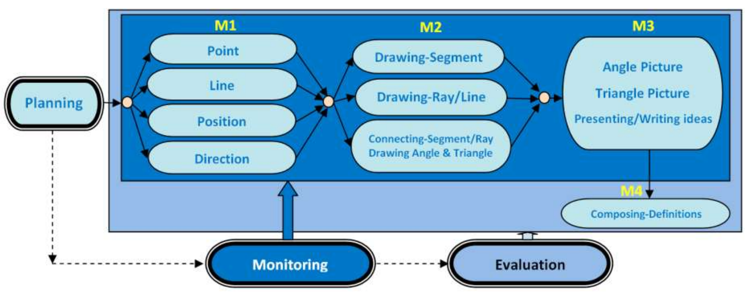

Figure 1. Metacognitive regulation scheme

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- *Drawing Geometry Figures Based on Recognizing Primitives Concepts (D)*

The use of figural and visual representations of a geometry concept is to help students understand the abstract ideas; however, there are many misconceptions cases due to lack of prior knowledge related to these geometry concepts development (Battista et al.,1991; Özerem, 2012). The learners were also making errors evoked by the concept image, i.e., the set of all the mental pictures associated in the student's mind with the concept name, together with all the properties characterizing them (Şahin et al., 2020). Consequently, these students fail in the construction of the definitions and misunderstanding the mathematical concepts. When the students' knowledge of geometry properties is incomplete, their concept image will differ from the required formal definition (Poon & Leun, 2016; Kembitzky, 2009). Thus, students' thinking approaches in defining a geometry concept should be emphasized to avoid memorizing image concepts. It also suggests that through a reflection and understanding guided by the educator, they should strictly depart from undefined geometry terms (primitive) and drawing geometry figures experiences to build a geometry concept in the standard definition form and other geometry terms in the broad sense.

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- *Writing Task to Present Ideas and Compose Definitions (W)*

Students' writing use to stimulate the dialogue for direct and indirect communication between the learners and the teacher in the teaching-learning process. Pugalee confirmed that writing helps mathematical thinking and supports learners in internalizing them of productive communication and relationship (in Urquhart, 2009). Through students writing, it could be known and assessed for the correctness of their already acquired mathematical. The studies discovered that the student's understanding levels improved, exchanged the reflections, and re-evaluated their answers. They are more developed in the competencies of reasoning, and ideas. They also improve to links between abstract mathematics and the context questioned. (Freeman et al., 2016; Barbara et al., 2016; Wilson & Nebraska, 2009). In short, it showed that there was a change from passive to active learners and the improvement of mathematical reasoning skills (Edig & Chavez, 2017).

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- *Interventing Students with Social Involvement (I)*

One of the most common conceptual changes using intervention and instruction strategies in the classroom was to induce cognitive conflict through presenting unusual (odd) facts or contradictory information (Limo´n, 2001). The mental conflict's primary goal is to make the learners disappointed with their present conception (Ozdemir & Clark in Kabaca et al., 2011). The educator can apply these strategies for students in both a lack of prior knowledge (missing knowledge) and the existence of incomplete knowledge or knowledge gap Chi, 2008; Chi, 2013). On the other side, Kowalski and Taylor's study suggested that the educator implement a critical thinking method to predict student misconceptions changing. Changing the students' misconceptions can happen for any abilities level and, significantly, to correct learners who think critically (Kowalski & Taylor, 2004). Using small group discussion, the educator may use Stepans' model for aiding students thinking contrary to their existing ideas. Through this group, the learners adjust to the new concept and resolve any current contradictions. Then they develop the concepts by connecting the thought learned in class with other associated concepts and ideas (in Sarar & Al-Migdady, 2014). In this action, the top priority of the educator's interventions and challenges is how to connect and correct the previously learned material of the student with their new knowledge.

- *Summary*

The MDWI approach aims to help the learners to exchange errors concepts with the right ideas. This approach, principally, encourages students to revise their existing preconception errors and accommodate the new idea. Hopefully, it can give learners opportunities to reflect and negotiate true mathematical meaning with their misconceptions and help them become stronger learners. They become much better prepared to exchange their incorrect geometry al concepts with correct concepts and proper decisions. Applying this MDWI strategy, we defragment and reconstruct or substitute the student’s misconception ideas and an understanding gap with natural ways and considering of following aspects. It involves the motivation and beliefs, prior knowledge, and cognitive engagement related to students; content knowledge, interests, and teaching strategies related to educator; the role of peers learning and learners-educator relation associated with the social context. The students’ eliminated errors correction includes misconception of generalization, concepts images, geometry features, and properties or others (Figure 2).

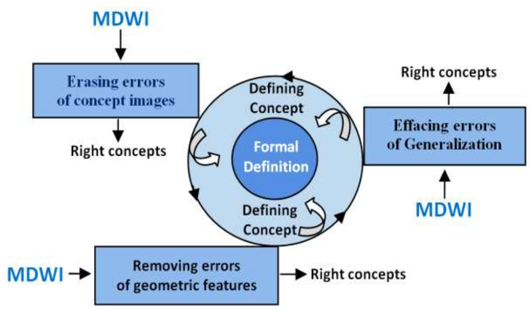

Figure 2. Errors elimination

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Step 3: Decision of Incorrect Concepts Exchange with Right Concepts

After MDWI treatment in Step 2, we evaluate the action results. When the student's works are considered satisfactory and coherent with the substitute explanatory concept, and the student believes that the substitute concept has a value for resolving problems, the educator can give them a chance to pursuing a new idea instructed. If not, the error remedial must restart as soon as possible.

Methodology

This study complied quantitative and qualitative descriptive research with the steps: gathering data, interpreting and analyzing data, and reporting the findings (Nassaji, 2015; Creswell, 2013). We used it because we wanted to understand in-deep students' misconceptions in defining an angle and the triangle before they attended a geometry course at the beginning of the first semester. Our investigation had the following three main objectives. (1). To identify students' common errors, the roots, and the causes of misunderstanding in defining the angle and the triangle. (2). To investigate the metacognitive regulation scheme in guiding and leading student thinking of geometry concepts. (3). To introduce the MDWI model for defragmenting, reconstructing, or linking students' knowledge from the incorrect geometry concepts to be correct concepts about angle and triangle definition.

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Participants and Times

The research involved 2 (two) mathematics education department students groups with 40 students per class from an excellent private university in Mataram, Indonesia. Both groups were undergraduate students. They graduated from public and private high schools and passed the national mathematics examinations included the geometry lesson. The research was held from March until December 2021.

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Instruments

The research instruments used tests in the form of an open-ended question about angle and triangle concepts to investigate students' errors and misconceptions of both geometry terms. The contents consisted of 3 questions associated with constructing these terms and writing into formal definitions. Each item of this test instrument was available for the students' opinion questions (Appendix). Referring to the students' test answer errors, we interviewed each student to discover their existing concept errors and misconceptions dealt with his/her formal definitions composed. Using the table, we classified the common error types, the roots, and the causes of misunderstanding (Table 1).

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Procedure

- *Identifying common errors and diagnosing the roots and the causes of misunderstanding*

Finding common errors and the roots of misunderstanding about defining angle and triangle were using open-ended questions test in Appendix A. The results of student work were evaluated and analyzed related to the occurrence of student answer errors. Then it was classified and coded from high common errors level (E1) to low errors (E5), as shown in Table 1. Discovering the roots of their misconception, ensuring learners mistake from the test answers (whether it was an error or misunderstanding), and improving awareness of collegers from the thinking, we interviewed them 1-1 through a personal approach related to their incorrect answers. The interview content was related to their experiences about the students' geometry pre-knowledge, ways of thinking, oral describing the definition of angle and triangle, and admitting his/her errors. The results of these activities were presented in Table 2. The learners of the understand and inadequate understanding in the category of minor misconceptions respectively treated by defragmenting and connecting the incorrect geometry concepts with the right ideas. Other, it treats using reconstruction actions as shown in Table 3.

- *Misconceptions' Correction Using MDWI and Concepts Exchange Decision*

The treatment for the minor error group's collegers was carried out individually following the metacognitive regulatory mechanism in Figure 1. In starting correction tasks of misconceptions, the students should find out all primitive terms and some supporting concepts used to define the angle and triangle (point, line, ray, segment, position, and direction). Using these elementary geometry objects, they should try to draw and present any angle and triangle shapes. Hereafter, they explained each construction process and wrote his/her result work in the formal definitions using their language. In this case, the intervention and instruction of the educator emphasized improving and revising the following knowledge aspects. (1) recognizing the pre-knowledge for defining the geometry concepts; (2) demonstrating the concepts' construction process with pictures; (3) presenting and writing this demonstrated concept idea; and (4) composing the formal definitions. The corrective function of the treatments is to fill the gap (hole) of students' conceptual understanding or revise and reconstruct the concepts' wrong parts.

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On the other hand, the treatment for learners from the acute error group remediated the same as those in the minor misconception group. Still, we divided them into some groups (3 students per group). The group function was to discuss and evaluate the alternative definitions that resulted from the group members. Hereafter, each group should finally produce some geometry alternatives definitions that were most suitable with the formal concept discussed. During this step, the educator used some counterexamples and cognitive conflict strategies to make collegers dissatisfied with their ideas. These ways are also applied to straighten students' thinking, guide, and help them to exchange the students' incorrect concepts with correct concepts. The scheme and results of these treatments are introduced in Table 4.

The educator evaluated the students' difficulties correction results of both minor and major misconception levels. If it considered that both groups' works were correct, clear, and coherent with the formal concepts, then they could pursue a new idea. If not, they should repeat in-deep.

Results

***Problem 1**: Identifying Common Errors and Diagnosing Roots and Causes*

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Based on the analysis results of student answers of the test items in the Appendix, it founded that thirty-seven students made errors in defining angle, and there were sixty-one students in explaining triangle. In this case, this research identified five types of students' common errors. First, collegers assumed that an angle is a point (**E1**). Second, they thought that an angle is the area part of a plane between two legs of this angle (**E**2). Third, collegers argued that an angle is a figure represented by two line segments combined at one endpoint of both line segments (**E3**). Fourth, they stated that a triangle is a part of a plane piece that forms the triangle (**E4**), and, fifth error, they concluded that any three line segments define a triangle (**E5**). On the other sides, in answering test item 1, there were 14 students (17.5%) of the error **E1**, 11 students (13.8%) of the error **E2**, and 12 students (15%) of the error **E3**. For the test items 2 and 3, respectively, there were 28 students (33.8%) of the error **E4** and 33 students (41.3%) of the error **E5**. Thus, the total errors of the test items 1, 2, and 3 were successively 37 students (46.3%), 28 students (33.8%), and 33 students (41.3%). The misconceptions frequencies (f) of these thirty-seven learners in solving geometry test items 1, 2, and 3 see in Table 1.

Table 1. Frequencies of students' misconception in understanding angle and triangle

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Problems	**Error 1 (E_1)**		**Error 2 (E_2)**		**Error 3 (E_3)**		**Error 4 (E_4)**		**Error 5 (E_5)**		**Total**	
	f	%	f	%	f	%	f	%	f	%	f	%
Test item 1	**14**	17.5	**11**	13.8	**12**	15	-	-	-	-	**37**	46.3
Test item 2	-	-	-	-	-	-	**28**	33.8	-	-	**28**	33.8
Test item 3	-	-	-	-	-	-	-	-	**33**	41.3	**33**	41.3

The errors' roots and causes of collegers misconceptions have resulted from the student works analysis and in-depth interviews. The interview content was related to their experiences about geometry pre-knowledge, ways of thinking, oral describing and writing the definitions of angle and triangle, and admitting errors. From the interview results, we also evaluated the missing and incomplete knowledge structures, the connection among the learned material of the collegers, and the logical consequences as follows.

Misconception 1: Over-specializing that an angle $\angle ABC$ is the point B.

Students who undergo this misconception could memorize that an angle definition $\angle ABC$ is the union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersected at the common endpoint B, but they mark point B as an angle (Figure 3a). Regarding results of interviews with the collegers of the misconceptions **E**1, did not generally understand some undefined terms and geometry's elementary objects used for defining rays, line segments, and drawing an angle. They also could not explain the relationship between these concepts to construct the angle. Moreover, they ignored the starting point position and the direction for a line ray and habited the writing angle symbol with only one capital letter. Due to the learners define the angle maybe with rote, consequently, they say that this angle is the point B.

Misconception 2: Over-generalizing that an angle is the area part of a plane bounded by two legs of this angle.

This misunderstanding appeared in the students' answers to test item 1 about the problem of angle. The learners already known an angle constructed by two rays intersect at the endpoint, but they stated an angle figure as the area bounded by their angles legs. For example, Figure 3b shows the work of a student who has this second misconception type. The in-deep interviews could inform the students' misconceptions from two causes. They did not understand that two rays met at the starting point would consistently result in the rays pieces connection (not a cut of plane), and they lacked the prior knowledge to differentiate between an angle and its measure. As a result, they have the misconception that an angle is the area part of a plane bounded by two legs of this angle.

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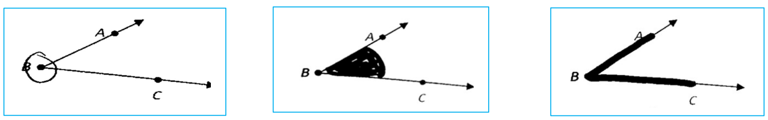
Misconception 3: An angle is a figure formed by two line segments that meet at one endpoint of the segments.

In the third misconception, the collegers recognized that the angle $\angle ABC$ was a union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ coincided at its starting point B, but they stated that an angle $\angle ABC$ was two line segments $\overline{BA}$ and $\overline{BC}$ that met at point B. For example, Figure 3c shows the works result of students that the line segments $\overline{BA}$ and $\overline{BC}$ as an angle. They argued an angle $\angle ABC$ was a set of points of line segments $\overline{BA}$ and $\overline{BC}$. Appertaining to the results of in-depth interviews, the collegers who make the misconceptions have not consistently differentiated between segments and rays to define an angle. They over-specialized this angle represented with three points and three capital letters. Because of this, the students said an angle was a figure formed by two line segments that meet at one endpoint of the segments.



(a) (b) (c)

Figure 3. Student work results in the completion of the test items 1

Misconception 4: A triangle is a part of a plane piece that forms the triangle.

Students consider a triangle as the interior of the triangle or the area bounded by the triangle. This misconception appeared in the students' works of test item 2 relating to the triangle concept. In this test item 2, the collegers who had a misunderstanding could define a triangle as a polygon of three sides but pointed out the graph that the triangle was an area bounded by the sides of the triangle (interior of the triangle). For example, Figure 4a shows the works of students experiencing the fourth misconception. Referring to the results of interviews, the collegers error **E**4 did not know that the merging three line segments at its endpoints for constructing triangle would produce three line-segments connection picture. They also used their primary school experiences in which a triangle was made from cutting paper through three noncollinear points. In consequence, these learners declared a triangle is a part of a plane piece that forms the triangle.

Misconception 5: Any three line segments define a triangle.

Students concluded that any three line segments can form a triangle. This misconception happened from test item 3 about three line segments as data for triangle sides. Learners understood that triangle sides had three line segments. Relating to the solution of test item 3, Figure 4b, the students explained that any three line segments could form a triangle, i.e., a right triangle or other triangles. Because of these triangle images and without examining the measure of these three line segments data, they made wrong conclusions.

(a) (b)

Figure 4. Students work results in the completion of the test items 2 and 3

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Appertaining to these students results of interview, the errors **E**1 to **E**5 found ten information about the roots of students' misconceptions. The list of these roots and causes of students' misunderstanding presents in Table 2.

Tabel 2. Common errors, the roots and the causes of students' misunderstanding

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Code	Common errors	The Roots and the Causes of Misunderstanding (Code)
E$_1$	An angle $\angle ABC$ is a point B.	1. The students do not understand some undefine terms and geometry's elementary objects used for defining rays, line segments, and drawing an angle (**E**$_{11}$). 2. They do not know the relation between these definitions to construct an angle (**E**$_{12}$). 3. They define the angle with the rote and habit the writing angle symbol with only one capital letter (**E**$_{13}$).
E$_2$	An angle is an area part of a plane bounded by two legs of the angle.	1. The learners do not understand that if two line rays meet at their starting point will consistently result in the rays' pieces connection (**E**$_{21}$). 2. They lack the prior knowledge to differentiate between an angle and its measure or the area between their angle legs (**E**$_{22}$).
E$_3$	An angle is a figure formed by two line segments that meet at one endpoint of the segments.	1. The learners can not consistently differentiate between segment and ray to define an angle (**E**$_{31}$). 2. They over-specialize an angle represented with three points and three capital letters (**E**$_{32}$).
E$_4$	A triangle is a part of a plane piece that forms the triangle.	1. The students do not know the merging three line segments at their endpoints for constructing a triangle will produce a line-segments connection picture (**E**$_{41}$). 2. They have a misconception from primary school experiences in which a triangle make from cutting paper through three noncollinear points (**E**$_{42}$).
E$_5$	Any three line segments define a triangle.	1. The students state the conclusion for defining a triangle using some triangle images without counting and comparing the length of three line segments (**E**$_{51}$).

Problem 2*: Misconceptions' Correction Using MDWI and Concepts Exchange Decision*

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In general, we found the sources of these misconceptions of collegers were the lack of prior knowledge or missing knowledge of geometry concepts (MK), the existence of knowledge gap or incomplete knowledge (IK), interpretation deviation of concept images (ID), feeble logical thinking (FT), and low connection of students knowledge (LC). These causes characterize the misconceptions in the following three types. In case the causes MK and IK, we call inadequate understanding with minor errors (10 students). For the causes ID and FT, we state inadequate understanding with major mistakes (18 students), another (LC), it calls almost understanding with minor errors (9 students). Resolving these misconceptions' causes, we introduce the approach to reconstruct thinking fragmentations of MK and IK, rearrange knowledge structures and logical thinking of ID and FT, and link knowledge of students LC.

For this solutions approach, we state, respectively, with the terms: defragmentation, reconstruction, and connection solutions as shown in Table 3.

Tabel 3: Treatment approaches of students' misunderstanding

Concepts Mastery Achievement	Number and kinds of Students Misconception and Treatment Types	
	Minor	**Major**
Inadequate understanding	10 Students of MK and IK *Defragmentation*	18 Students of ID and FT *Reconstruction*
Almost understanding	9 Students of LC *Connection*	-

This section reports the errors correction of collegers using the MDWI approach. It cures the students' misunderstanding roots in Table 2 for their errors cases in Table 1. The stages of the MDWI were as follows (Table 4). Implementing the metacognitive regulation scheme presented in Figure 1, the instructor directed the collegers to learn primitive concepts of the angle or triangle (activity M1) and design figures of the angle or triangle (activity M2). Evaluating these students' activities was focused on drawing an angle or a triangle idea connected with primitive concepts. The stage of activities M1 and M2 is called the drawing concept (D). Then, the collegers presented pictures and wrote geometry ideas based on prior knowledge (activity M3) and composed formal definitions (activity M4). The instructor helped the learners to recognize the geometry characteristics and connections of the angle and triangle elements, the logical thinking for constructing an angle or triangle and write the definitions of angle and triangle through the pictures. The stage of activities M3 and M4 is called the writing task (W). During the learning activities D and W, the instructor gave interventions and instructions to develop the learners' knowledge structure and induce cognitive conflicts. These cognitive conflicts are designed to resolve the causes of students' misunderstanding MK, IK, ID, FT, and LC. The intervention actions of the cognitive conflict, i.e., respectively, marked by the code $\mathbf{I}_{MK}$, $\mathbf{I}_{IK}$, $\mathbf{I}_{ID}$, $\mathbf{I}_{FT}$, and $\mathbf{I}_{LC}$ presented in columns 4-7 in Table 4. Students with minor misconceptions were individually treated, in contrast, to the major misconceptions, remedied in group of five students. Assessing students' achievement with scores interval 0–100 and in-depth interview, the treatments found the average result scores shown in column nine of Table 4.

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Some cognitive conflicts examples associated with the treatments of students' misunderstanding MK, IK, ID, FT, LC in Table 2 and often used by the instructor for interventing collegers in this research were as follows.

1. An angle $\angle ABC$ is just a point B that is undefined geometry object; versus a set of points consisting of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersects at the endpoint *B*.
2. An angle $\angle ABC$ is a measure of arc degree or an area between two angle legs $\overrightarrow{BA}$ and $\overrightarrow{BC}$; contra to the joint of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ coincides at the point *B*.
3. An angle $\angle ABC$ is a joint of two line segments $\overline{BA}$ and $\overline{BC}$ meet at the endpoint *B*; against the union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersects at the endpoint *B*.
4. An angle $\angle ABC$ is just three points *A*, *B*, and *C*; versus a union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ meets at the point *B*.
5. The joining of three line segments $\overline{BA}$, $\overline{BC}$, and $\overline{AC}$ at their endpoints *A*, *B,* and *C* will form a cut of a plane *ABC* called a triangle *ABC*; contra to they produce a picture of three-line segments called a triangle *ABC*.

6. Every three line segments $\overline{BA}$, $\overline{BC}$ and $\overline{AC}$ can form a triangle ABC; contra to the line segments of the measures $\overline{BA}$ = 3 cm, $\overline{BC}$ = 5 cm, and $\overline{AC}$ = 15 cm will not construct a triangle ABC.

Tabel 4: Errors treatment using MDWI

Errors Roots	Students Number	Treatment Types	Metacognitive Regulation (M)				Correct Students Number	Average Result Scores (0 – 100)
			Drawing Concept (D)		Writing Task (W)			
			M1	M2	M3	M4		
E_{11}	**7**	Defragmentation, Reconstruction, & connection.	I_{MK}	I_{IK}	I_{ID}	I_{FT}-I_{LC}	**5**	**71.43**
E_{12}	**4**	Reconstruction.	-	-	I_{ID}-I_{FT}	I_{FT}	3	**75.00**
E_{13}	**3**	Reconstruction & connection.	-	-	I_{ID}	I_{FT}-I_{LC}	2	**66.67**
E_{21}	**6**	Reconstruction	-	-	I_{ID}	-	4	**66.67**
E_{22}	**5**	Defragmentation & reconstruction.	I_{MK}-I_{IK}	I_{ID}	I_{ID}	-	4	**80.00**
E_{31}	**7**	Defragmentation & reconstruction.	I_{MK}	I_{ID}	I_{ID}	-	5	**71.43**
E_{32}	**5**	Reconstruction.	-	-	I_{ID}	I_{FT}	4	**80.00**
E_{41}	**17**	Reconstruction & connection.	-	-	I_{ID}-I_{LC}	-	13	**76.47**
E_{42}	**11**	Reconstruction.	-	-	I_{ID}	-	8	**72.73**
E_{51}	**33**	Reconstruction & connection.	-	-	I_{ID}	I_{LC}	24	**72.73**
Total	**98**	**Number of Interventions**	**4**	**3**	**12**	**7**	**72**	**73.31**

Discussion

Based on Table 1, it informs that the only 43 students (53.7%) are correct in understanding the concept, and 37 students (46.3%) can not define the angle properly. On the other hand, it finds 33 of the 61 students (76.3%) who fail in understanding the triangle idea. Then, the only 19 students (23.7%) are successful in clarifying the concepts. Thus, there are no more than 54% students can correctly define both angle and triangle terms.

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Evaluating the roots and the causes of misunderstanding in Table 2, these can identify the impediments and feebleness of students. The collegers do not know the primitive terms, the function, and the role of the ray and line segment in defining an angle or triangle. In this case, they learn the geometry concepts and definitions partially. These results were relevant to the studies of Ozkan et al. (2018), Al-Khateeb (2016), and Özerem (2012), who informed that one of the misconceptions causes was the lack of prior knowledge and insufficient students' knowledge of the geometry concept. In addition, it made misconceptions in reasoning and basic operation mistakes. Besides, the mathematical material was, generally, interconnected. So, this study should eliminate the previous errors before introducing a new issue. After that, the misconceptions may happens to some images interpretation errors in the geometry concepts understanding. For example, the collegers undergo the visual deviation between a point and an angle, an angle measure and an angle, and between a triangle and a plane cut triangle. Then, they have trouble reasoning for constructing an angle and triangle using the rays and line segments. There is limited understanding of geometry relations between the points, lines, rays, and line segments to determine an angle and triangle. These finding results is inline with the studies of Poon & Leun (2016), Biber et al. (2013), Cunningham & Roberts (2010), and Gal & Linchevski (2010) who found that learners faced difficulties in selecting the characteristics of figural elements relevant to the concepts. Due to the students focused only on the physical shapes and the geometry images rather than identifying the essential geometry properties of represented figures and fundamental logical reasoning abilities.

Handling the students' misconceptions as presented in Table 4 shows that it is necessary for at least 26 interventions provided by the instructor. Implementing these strategies was based on the metacognitive regulation scheme (M) that focus on solutions M3 (12 interventions), M4 (7 interventions), M1 (4 interventions), and M2 (3 interventions). The treatment actions resolve the students' difficulties with the reconstruction approach at least seventeen times, the connection way four times, and the defragmentation technique five times.

Referring to the frequencies amount of the interventions $\mathbf{I}_{ID}$ (12 times) and $\mathbf{I}_{FT}$ (5 times), it can conclude that, in general, students' misconceptions are the interpretation deviation of the image concept to define angle or triangle. The source of this main problem is that the collegers fail to recognize the geometry objects for drawing the angle or triangle (activity M2) and flunk to clarify, logically of how to construct this angle or triangle by using these objects (activity M3). Because this activity is not passed well by collegers, it is naturaly misperceive that an angle is a point, three points, or a combination of line segments. Furthermore, they cannot distinguish that the triangle is a combination of three line segments, but they perceive it as the interior area of the triangle. The students make a wrong generalization (over-specializing and over-generalizing). Then, the frequencies $\mathbf{I}_{MK}$ (3 times) and $\mathbf{I}_{IK}$ (2 times) indicate that, for activity M1, the collegers do not have much knowledge about the geometry elementary objects (primitive terms) and definitions to construct the angle and triangle. Hence, collegers assume that an angle represented with the legs of line segments or rays pictures is equal. They also consider the angle as three points (concept images errors). On the other hand, the frequencies $\mathbf{I}_{LC}$ (4 times) signify that, for activity M4, the learners are error to describe and define an angle and triangle with their words, due to, they fail to connect the features of points, line segments, and rays to construct the angle and triangle. As a result, they define the angle with rote or using some triangle images.

From Table 4, we can review the MDWI method for correcting the student misconceptions as follows. The defragmentation, reconstruction and connection treatments for the error root $\mathbf{E}_{11}$ using interventions $\mathbf{I}_{MK}$, $\mathbf{I}_{IK}$, $\mathbf{I}_{ID}$, $\mathbf{I}_{LT}$ and $\mathbf{I}_{FC}$ in metacognitive regulation activities M1, M2, M3, and M4 give the score of 71 (very good). Moreover, the defragmentation and reconstruction treatments $\mathbf{E}_{22}$, and $\mathbf{E}_{31}$ with interventions $\mathbf{I}_{MK}$, $\mathbf{I}_{IK}$, and $\mathbf{I}_{ID}$ in metacognitive regulations M1, M2, and M3 remedied the collegers in identifying the angle fitures with the average correct result score of 80 (excellent). In case the interventions $\mathbf{I}_{MK}$ and $\mathbf{I}_{ID}$, they achieve the average score of 71 (very good). The overall reconstruction and defragmentation treatments could avoid significantly the incomplete and missing knowledge of students, and the interpretation errors of the angle concept images.

The reconstruction treatments $\mathbf{E}_{12}$, $\mathbf{E}_{21}$, $\mathbf{E}_{32}$, $\mathbf{E}_{42}$ with intervention I_{ID} in activities metacognitive regulation M2 and M3 declined the students errors in drawing, recognizing features, and explaining ideas in the constructing angle and triangle. The students could attaint the average correct result score of 70 (very good). Besides, in the interventions $\mathbf{I}_{ID}$ and $\mathbf{I}_{FT}$, they achieved the average score of 78 (very good). These reconstruction treatments, in

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general, improved their skills in demonstrating figures, describing the ideas, and making generalizations to define the angle and the triangle.

The reconstruction and connection treatments $\mathbf{E}_{13}$, $\mathbf{E}_{41}$, and $\mathbf{E}_{51}$ with interventions I_{ID} and I_{LC} in metacognitive regulation activities M3 and M4 remedied the learners' errors of the concept images and the interconnecting of geometry concepts. The students could achive the average correct result score of 75 (very good) in the angle and triangle comprehension. For the interventions $\mathbf{I}_{ID}$, $\mathbf{I}_{FT}$ and I_{LC}, it finds the score of 67 (good). These treatments could increase the collegers skills in reconstructing and connecting previous geometry concepts to define the angle and triangle.

Based on the remedies results score of 73 (very good), and the students of the correct answers of 72/98 = 73 %, it could state that the overall MDWI treatment approach could help the collegers to exchange their misconceptions in defining angle and triangle from misconception concepts to be correct concepts. However, it suggested that, in constructing the definitions of geometry ideas, the students have to custumize using this metacognitive regulation scheme. If not, otherwise it is worried that they will only rote these concepts. These students' corrections results in averages come across that their works have been correct and coherent with the formal definitions, and each student's score was more than 60. Thus, they could pursue a new learning topic. To sum up, these interventions scheme and MDWI method resolve the student errors in generalization, concept images, and connection incompetencies of geometry features that were discussed by Gutiérrez & Jaime (1999), Özerem (2012), Poon & Leun (2016), Ozkan et al. (2018), and Şahin et al. (2020).

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Conclusion

The common errors occurred in defining angle and triangle of mathematics education collegers, i.e., the assumption and generalization that angle is a point, an area part of a plane between two legs of the angle, and a union of two line segments that meet at one endpoint of the segments. Then, they also made the misconceptions that a triangle is a part of the plane piece forms the triangle, and any three line segments define a triangle. The roots of these errors, generally, are of basic ideas. They do not know the primitive terms, the function, and the role of the ray and the line segment for defining an angle or triangle. Moreover. their understanding of geometry concepts and definitions is still partially. The students commonly fail in concept images interpretation, logical thinking, and knowledge connection needed to draw, construct, and write the definitions of angle and triangle.

Errors correction using the MDWI treatment approach guides the collegers to learn primitive concepts, draw the geometry idea connected with primitive ideas, present and write these pictures into formal definitions. During this learning-teaching process, the instructor gives interventions and instructions to develop their knowledge structure, induce cognitive conflicts, and resolve the causes of their misunderstanding relating to the generalization errors, interpretation deviation of concept images, and connection incompetencies of geometry features. In most cases, these treatments effects empowered the collegers to decline and reduce their errors in recognizing the geometry objects and generalization, interpretation concept images, and connection of geometry features.

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Recommendations

Considering errors correction is rarely in defining geometry objects, this MDWI treatment provides a guide for cure the learners of the geometry concepts misconception step by step based on the metacognitive regulation. Inline with the university learners' errors amount of

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these generalizations and concept images, the treatments M2 and M3 are very effective for helping them to avoid the misunderstanding of interpretation deviation and logical thinking of concept images and geometry features. Based on this research results, it is suggested that future research is tried the junior and high school students.

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Limitations

This research was conducted on students elected from an excellent private university in NTB Province, Indonesia. Thus, the generalization of the results in this research has limitations. Another limitation of the study was that it only focused on defining the terms of geometry concepts.

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Appendix

Instruments for Identifying Angle and Triangle Misconceptions and Types of Problems

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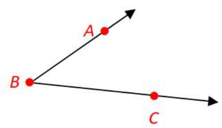
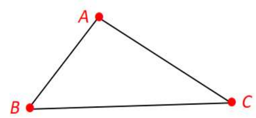
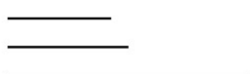
Problem	Problems Types
1. Consider the points *A*, *B*, and *C* in Figure 1. Give a mark using a colored pen, which is a part of Figure 1 called an angle?. Give your reason in detail!  **Figure 1.** Angle	Angle Problem
2. Let three points *A*, *B*, and *C* in Figure 2. Give a mark using a colored pen, which is a part of Figure 2 called a triangle?. Give your reason in detail!  **Figure 2.** Triangle	Triangle Problem
3. Given any three line segments in Figure 3. Can these line segments form a triangle and explain why!  **Figure 3.** Three line Segments	Line Segments Problem on Triangle

Identifying and Correcting Students' Misconceptions in Defining Angle and Triangle

Abstract: Misconceptions are one of the most handicaps to comprehend in learning mathematics. This research aimed to investigate the students' common errors and the misconception caused by ~~causes of~~ students in defining the angle and triangle. Moreover, we studied metacognition/drawing/writing/intervention (MDWI) strategy to change the students' understanding of concepts from the wrong concepts to be the correct ideas. To achieve this goal, it used a research design. It identified and resolved the errors in defining the angle and triangle of the first-year undergraduate students of the mathematics education department from an excellent private university in Mataram, Indonesia. The steps were as follows. Open-ended questions tests instrument and in-depth interviews were used to identify the errors, the roots, and the causes of students' misunderstanding. After that, MDWI approach was used to introduce the way to correct these errors. It was found that students generally failed in concept images interpretation, logical thinking, and knowledge connection needed to define the angle and triangle. The MDWI remedied the misconceptions of generalization, concept images errors, and connection incompetency of geometry features.

Keywords: *Angle and triangle, cause, common errors, misconception correction.*

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Introduction

Reports from the international organization and the Indonesian ministry of education describe that secondary school students' mathematics achievement in Indonesia tends to decline in recent years. These include abilities in defining concepts, measurement (quantity), relationships, logical thinking, and computing. The results of tests and evaluations from the Program for International Student Assessment (PISA) between ~~along~~ the years 2015 and 2016 undertaken by the Organization for Economic Cooperation and Development (OECD, 2016) are classified as low completeness. The tested material also includes geometry. It means that students have not fully mastered geometry's main subjects, including quantity, relationship, and uncertainty (Lemke et al., 2004). In 2019, the mathematics score ~~score's~~ national exam of junior high school students failed, namely in the position 45 of score interval 0-100. Besides, for senior high school level, the student's achievement to answer all given questions of the geometry and trigonometry test with correct responses category was only 37% in 2017; and 34% in 2018. Mastering the students' mathematics was still not good, i.e., in average score 45 of interval 0-100 (Ministry of Education and Culture of the Republic of Indonesia, 2019). These fail cases will potentially repeat in future studies of the students, particularly in the first year of an undergraduate program.

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The students' weak understanding of geometry definitions and concepts as the geometry pre-knowledge will affect low future mastering geometry, difficulties, and failure. One factor in which the students can happen is the geometry misconception when their concentration only focuses on the physical shape and the geometry images rather than identifying the essential geometry properties of represented figures (Biber et al., 2013; Poon & Leun, 2016). Based on the geometry questions to the students about the definitions of angle, measure, and shape, this study reported a lack of students' background knowledge that makes many learner misconceptions in reasoning and basic operation mistakes (Özerem, 2012). These facts indicate that the students have to develop their understanding of geometry concepts and some related abilities.

A theoretical and passive teaching approach that provides very few visuals and tends to ask the students to memorize, does not guarantee that students can master the definitions and geometry concepts. It was reported that the images of geometry objects could be used to illustrate the relationship between one and another concept. These images would help the students to understand the abstract ideas and motivate the students to acquire the needed knowledge. Moreover, concept images affected the students in transforming the situation model into a mathematical model (Battista et al., 1991; Phillips et al., 2010; Şahin et al., 2020). To discover the properties of geometric figures should be a process directed by definitions, axioms or theorems (Karpuz & Güven, 2022). Besides, stimulating and challenging geometry concepts is required that students will be able to improve their understanding. A lack of a formal definition could cause problems for students as they will unable to test their own conception of the idea against the formal theory (Hogue & Scarcelli, 2020). Several studies (Cunningham & Roberts, 2010; Gal & Linchevski, 2010; Özerem, 2012; Ubi et al., 2018) informed that students failed to complete dimensional deconstruction of images to obtain mathematical properties. They found it difficult to determine the characteristics of figural elements relevant to the concept, transformations and construction, and 3-D shapes. This misunderstanding can occur because of the teachers themselves or their environment, i.e., the proficiency and inadequate book facilities. It also appeared due to incomplete reasoning and wrong intuition (Kamid et al., 2020). . Although the teachers' role in developing knowledge is essential, they also need to play an active role in dealing with their misunderstanding problems. Through metacognitive activities, such as their own written work error analysis, students can find and try to align their conceptual inconsistencies with more formally accepted mathematical constructs (Tirosh, 1990, as cited in Kembitzky, 2009). This conceptual change needs some strategies and metacognitive skills. Referring to Stepans' model of conceptual change, to change the alternative concepts of students needs the nature of learning tasks that can help students exchange their understandings with the right ideas. The nature of the learning environment can involve social dialogs and negotiations among students (Sarar & Al-Migdady, 2014).

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These studies provide information that the teachers' role in developing knowledge is essential. Challenging concept achievement is required for the students to improve their ideas. Using the image of geometry objects in teaching-learning helps to understand abstract concepts and acquire the needed knowledge. However, it still finds some students difficulties in understanding geometry concepts. The students only focus on the physical shape and lack of students' prior knowledge of geometry. They also fail to obtain mathematical properties through the image media. Meanwhile, to help students change the misconception understanding, it needs some strategies and metacognitive skills. Because of these geometry learning impediments and limitations, we have to make aware and strengthen the conceptual understanding of the error correction of geometry lessons. Consequently, it needs a new learning strategy to improve the correct geometry concepts and reasoning. For these reasons, this research will identify the errors and the cause of misconceptions and strategize to reconstruct the students' misconceptions of geometry concepts.

Literature Review

Metacognition is vital in supporting the performance of cognitive tasks in mathematics learning. Some research results reported that metacognitive understanding includes aspects of cognition's knowledge and cognition's regulation. This knowledge contains the cognitive abilities, processes, resources, and the influence of a person, task, or strategy factors on performance (Brown et al., as cited in Garofalo & Lester, 1985, p.164). The regulation of metacognition is concerned with the strategic decisions' activities in a course via cognitive tasks. These activities consist of planning studies, monitoring processes, evaluating and revising the outcomes. Sternberg (2002) stated that metacognition is diverse. It includes both understanding and control of cognitive processes that include planning, monitoring, and evaluating activities. This understanding process, of course, must be effective action. It must also be remembered that metacognition interacts with many other aspects of the student, i.e., abilities, personality, and learning styles. Magiera and Zawojewski (2011) used metacognitive awareness, regulatory, and evaluative approaches for small-group students. They identified and characterized the social-based and self-based contexts related to their metacognitive activities in mathematical modeling learning. The metacognition approach supports the learning process in mastering concepts and geometric reasoning (Nahmias & Teicher, 2021; Wonu & Charles-Ogan, 2017). The application of metacognition regulation expects to prevent students from thinking that they only memorize concepts. It can be a tool to make students aware and correct their reasoning weaknesses in mastering the concept of geometry. In the learning process, the teacher can use it to determine the weak points of student reasoning, develop the strategies for learning, and determine some stages of student error correction.

Visual geometry objects help build people's experiences, beliefs, and understanding of an item through a cognitive process. Logically, the images of geometry objects are usable to explain the relationship between one and another concept. The efficacy of visualization will help understand concepts and support students to acquire educationally the required knowledge (Phillips et al., 2010). On the other sides, as a tool in learning geometry, images of geometry objects are widely used to explain definitions and concepts of geometry, but some students still have misconceptions. They fail to match both concept's formal definition and the geometrical figures (Vinner & Hershkowitz, 1980). Berthelot & Copy (as cited in Poon & Leun, 2016) stated that one of the students' misconceptions factors in geometry learning is an incapability to identify various shapes (symbolic, visual, etc.) of the same geometry concept. Özerem (2012) found the student feebleness of measures, angles and shapes, transformations and construction, and 3-D shapes. This is a real challenge for university educators in which they generally refuse to be corrected. Providing personalized interventions to help students resolve misunderstandings in this context is a difficult challenge. Educators must work with their students to identify, recognize, and correct commonly held misconceptions to attain the best learning outcomes. Any student misconceptions critically need to be evaluated, revised, and changed with information consistent with the accepted concepts (Verkade et al., 2016). Generally, there were five causes of errors: language misconceptions, spatial information difficulties, deficient mastery of prerequisite skills, facts, and concepts; fallacies of thinking; and the application of irrelevant rules or strategies (Radatz, 1979, as cited in Kim, 2011). Ay (2017) reviewed the errors. It states that apart from the test, one of the most appropriate ways for detecting these students' misconceptions is collecting qualitative data through interviews or observations. These facts provide in-depth information about the students' difficulties in learning. However, the researchers did not prefer going on further steps. Research about the remedial misconceptions of mathematics, particularly in understanding geometry concepts, is still limited.

Teacher's teaching and intervention strategy can make a difference in students' comprehension, which is essential in instructional practice and student learning. In geometry learning, Lim (2011, as cited in Luneta, 2015) states that the information's the information's communication at the different level of reasoning among the teacher and student becomes a common cause of misconception. When teachers explain different geometry thinking levels to students, the concepts are not fully understood or acquired. Teachers must know their students' level of geometrical understanding. Battista et al. (1991) reported that developing the students' meaningful comprehension of geometry concepts requires an appropriate instructional task and assessment in teaching and learning geometry. Clarke et al. (1993, as cited in Kembitzky, 2009) found that writing allows a teacher to see the kind of thinking and understanding that is not easy and accessible via the computational and proficiency test. Teachers can examine the process of sense-making when students explore and work with mathematics. Therefore, teachers' intervention and students' writing assignments will direct the achievement of conceptual understanding following curriculum objectives. It can help the students to use previous experiences correctly and providing a new comprehension of the shortcomings of prerequisite material that students do not yet have and avoiding understanding concepts via rote, but by understanding processes.

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In summary, the discussion of these research findings provides essential clues for resolving the misconception problems of the students. The metacognition approach supports the learning process in mastering concepts and geometric reasoning. It can also be a tool to make students aware and correct their reasoning weaknesses in understanding geometry concepts. Using geometry images can help students identify, recognize, and remedy misconceptions to attain the best learning outcomes. Writing geometry ideas can employ to see the kind of students' thinking and understanding in which the lecturers can direct the achievement of conceptual understanding following instructional objectives. Taking in to account this thought, we conducted this research.

Problems and Purposes of Research

Students' ability to define geometry concepts is one of the main goals of achieving the first-year undergraduate program's mathematics competencies. Unfortunately, using geometry figures and tools to support the students in understanding the concepts and the definitions invoke some errors and misconceptions. They are difficult to determine the characteristics of figural elements relevant to the idea and often fail to arrange the words to construct the alternative definitions. To change the students' incorrect concepts with the right ideas, of course, needs the natural and suitable learning methods. This research addressed to answer the following problems.

1. What common errors and roots of misconceptions do frequently occur in defining the angles and the triangles of mathematics education students of the undergraduate program?
2. How can strategies resolve and exchange the students' misconceptions about definitions of angle and triangle from misconception concepts to be correct concepts in the teaching-learning process?

This research aimed to investigate the students' common errors and the causes of misconceptions in defining the angle and the triangle. Moreover, it introduced the strategies for changing the students' understanding from the wrong concepts to be the correct concepts about angle and triangle.

Framework of Research

Stage 1: Identification of Misconception

Students' misconceptions of geometry concepts can occur due to many factors such as student experiences and learning approach, teacher roles, and facilities (Cunningham & Roberts, 2010; Gal & Linchevski, 2010; Özerem, 2012; Poon & Leun, 2016). The studies found that some students have errors related to a deficiency of understanding geometry figures and insufficient knowledge of proof's importance. They fail to state the particular polygons and features of the polygons (Alamian et al., 2020; Cirillo & Hummer, 2019; Herholdt & Sapire, 2014; Junus, 2018). Students' difficulties in understanding geometry concepts are various. To decide the strategy for the remedial process, this research needs to identify the students' misconceptions types.

Stage 2: Misconceptions Diagnosis

Some research studies reported the causes of concepts misunderstanding should be eliminated. Because mathematical materials are generally interconnected, the students' misconceptions about previously discussed topics should be resolved before introducing a new issue (Al-Khateeb, 2016; Ozkan et al., 2018). In this research, the purpose of students' misconceptions diagnostic is to identify their error levels and find out the causes and the roots of their difficulties in teaching-learning geometry. Besides, it is also meant to investigate the weaknesses of the teaching approach and the facilities used by educators. Using the interview method for diagnosing the errors can provide recent information on student weaknesses and flexibility of examining; meanwhile, open-ended test methods will support the students' chance to write their answers in their own words. They will probably give some new valuable responses (Gurel & Eryılmaz, 2015).

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Stage 3: Strategy and Correction of Misconceptions

There are three treatment steps to conduct the strategy and errors corrections i.e., student awareness; defragmentation, reconstruction, and geometry concepts connection of students' knowledge; revision and decided to exchange from the wrong to the right ideas. The ways are as follows.

Step 1: Student Awareness of Errors and Difficulties

The first step to change the misconception is to make the student aware that there is an error from the beginning. Removing the students' impediments must crucially come from themselves, including the held beliefs and prior knowledge (Verkade et al., 2016). Kruger and Dunning (2009) also warned that students who do not know their abilities would suffer a double burden; first, they only reach wrong conclusions; second, they are problematic to raise metacognitive skills to realize it. A study by Taylor and Kowalski (2004) informed that the power of belief is a significant transitional variable that may turn over in one's mind the change process. Furthermore, Hughes et al. (2013) concluded that it is easier for students to disregard, reinterpret, or refuse new information rather than to change their beliefs. In this study, changing students' conceptual was carried out with motivational activities, namely being dissatisfied with their previous views and providing a clear alternative explanation and rationale to the students.

Step 2: Defragmentation, Reconstruction, and Connection Treatments of Incorrect Concept

- *Metacognitive Regulation*

Supporting and guiding student errors correction is needed via the learning process. Applying the metacognitive processes approach: planning strategies, monitoring, and evaluation will expect to resolve the students' wrong concepts through defragmentation or substitution, reconstruction, and connection processes of thinking structures (Artzt & Thomas, 1998; Garofalo & Lester, 1985). These processes are, respectively, to reconstruct thinking fragmentations of misconception, link knowledge and rectify an idea error minor, rearrange their knowledge structures and the logical thinking error. In this research, it conditions the students to be aware of the troubles as they know them from the first place, then, the educator brings the students to be active thinking to change the wrong concept with the right ideas. The metacognitive regulation guides the students and instructors to design, control, evaluate, correct the understanding of geometry al features and structure the words for defining angles and triangles.

The scheme of metacognitive regulation for this research involves the students' understanding of the geometry prior knowledge for constructing an angle and triangle, i.e., point, line, position, and direction. Using undefined terms, the students must recognize the formal definitions of a line segment and a ray. Drawing, connecting, or combining among the points, the lines, the segments, and the rays, they have to try to find the shapes of the angle and the triangle. Using these constructed pictures, the students practice to explain and compose the definitions with their words. In short (Figure 1), they must resolve four treatment stages in the process of correcting misconceptions, i.e., recognizing primitive concepts and adding their pre-knowledge related to the defined geometry concepts (M1); drawing and demonstrating the concepts' figures (M2); presenting and writing geometry ideas connected with the prior knowledge (M3); composing the formal definitions (M4). These treatment series help the students to identify, reflect, evaluate, and correct the misconceptions of angle definition and triangle in natural ways. From the teacher's side, it can quickly detect and correct the students' errors from these stages. Thus, teachers' instruction and intervention for student error correction will be more focused and effective.

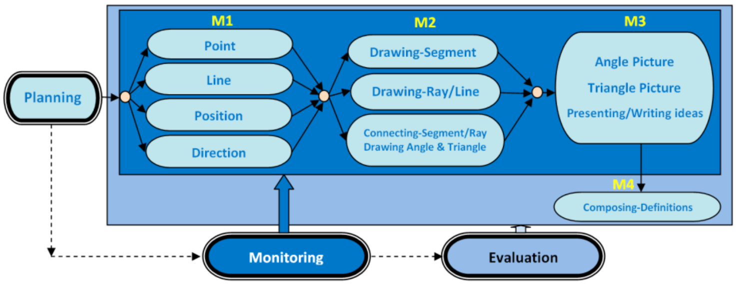

Figure 1. Metacognitive Regulation Scheme

- *Drawing Geometry Figures Based on Recognizing Primitives Concepts*

The use of figural and visual representations of a geometry concept is to help students understand the abstract ideas; however, there are many misconception cases due to lack of prior knowledge related to these geometry concepts development (Battista et al., 1991; Özerem, 2012). The students made errors evoked by the concept image, i.e., the set of all the mental pictures associated in the student's mind with the concept name, together with all the properties characterizing them (Şahin et al., 2020). Consequently, these students failed in the construction of the definitions and misunderstanding the mathematical concepts. When the students' knowledge of geometry properties is incomplete, their concept image will differ from the required formal definition (Kembitzky, 2009; Poon & Leun, 2016). In this case, the researches use students' thinking approaches in defining a geometry concept that were emphasized to avoid memorizing image concepts and guided by the educator. They have to strictly depart from undefined geometry terms (primitive) and drawing geometry figures experiences to build a geometry concept in the standard definition form and other geometry terms in the broad sense.

- *Writing Task to Present Ideas and Compose Definitions*

In this research, students' writing is used to stimulate the dialogue for direct and indirect communication between the students and the teacher in the teaching-learning process. Pugalee confirmed that writing helps mathematical thinking and supports students in internalizing them of productive communication and relationship (as cited in Urquhart, 2009). Through students' writing, it could be known and assessed for the correctness of their already acquired mathematical. The studies discovered that the student's understanding levels improved, exchanged the reflections, and re-evaluated their answers. They are more developed in the competencies of reasoning, and ideas. They also improved to link between abstract mathematics and the context questioned. (Barbara et al., 2016; Freeman et al., 2016; Wilson & Nebraska, 2009). ~~In short, it showed that there was a change from passive to active students and the improvement of mathematical reasoning skills (Edig & Chavez, 2017).~~

- *Interventing Students with Social Involvement*

One of the most common conceptual changes using intervention and instruction strategies in the classroom was to induce cognitive conflict through presenting unusual (odd) facts or contradictory information (Limo´n, 2001). The mental conflict's primary goal is to make the students disappointed with their present conception (Ozdemir & Clark, 2007, as cited in Kabaca et al., 2011). The educator can apply these strategies for students in both a lack of prior knowledge (missing knowledge) and the existence of incomplete knowledge or knowledge gap Chi, 2008; Chi, 2013). On the other side, Kowalski and Taylor's study suggested that the educator implements a critical thinking method to predict student misconceptions changing. Changing the students' misconceptions can happen for any abilities level and, significantly, to correct students who think critically (Kowalski & Taylor, 2004). Using small group discussion, the educator may use Stepans' model for aiding students thinking contrary to their existing ideas. Through this group, the students adjust to the new concept and resolve any current contradictions. They then develop the concepts by connecting the thought learned in class with other associated concepts and ideas (Stepans, 2011, as cited in Sarar & Al-Migdady, 2014). In this study, the main priority of the educator's interventions and challenges is how to connect and correct the previously learned material of the student with their new knowledge.

Step 3: Decision of Incorrect Concepts Exchange with Right Concepts

After treatment in Step 2, we evaluate the action results. When the student's works are considered satisfactory and coherent with the substitute explanatory concept, and the student believes that the substitute concept has a value for resolving problems, the educator can give them a chance to pursuing a new idea instructed. If not, the error remedial must restart as soon as possible.

Methodology

This study design complied with quantitative and qualitative descriptive research. The steps of the method were as follows: gathering data, interpreting and analyzing data, and reporting the findings (Creswell, 2013; Nassaji, 2015). This approach was utilized because it helped us understand students' in-deep misconceptions in defining an angle and the triangle before they attended a geometry course at the beginning of the first semester. The investigation had the following three main objectives. (1) To identify students' common errors, the roots, and the causes of misunderstanding in defining the angle and the triangle. (2) To investigate the metacognitive regulation scheme in guiding and leading student thinking of geometry concepts. (3) To introduce the strategies for defragmenting, reconstructing, or linking

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students' knowledge from the incorrect geometry concepts to be correct concepts about angle and triangle definition.

Participants and Times

The research involved two mathematics education department groups of students with 40 students per class from a private university in Mataram, Indonesia. Both groups were undergraduate students. They graduated from public and private high schools and passed the national mathematics examinations included the geometry lesson. The research was held from March until December 2021.

Instruments

The main instruments of researchers and auxiliary instruments of lecturers were used in this research. The main instruments were used for observing, collecting, analyzing, and interpreting research data. Then, the auxiliary instruments made the definition questions of angles and triangles and composed the unstructured interview guidelines validated by two experts. The researchers used open-ended question tests instrument about angle and triangle concepts to investigate students' errors and misconceptions of the geometry ideas. The test instrument consisted of three questions based on the content areas related to the prior knowledge, drawing the shapes, writing ideas, and defining angles and triangles. The reliability of this instrument was evaluated using the test-retest method for the test scores interval 0-100 in the range of 20 days from 52 students (two classes) on the students' previous batch. Calculating the correlation of the successive test-retest results through the formula of Pearson's product moment correlation coefficient r, it obtained a considered good coefficient with the reliability index $r \geq 0.7$ at 0.05 significant. The validity of each test item was determined by the correlation value r between the item score values and the total item score. It was found the values r in interval $0.6 \leq r \leq 0.8$. On the other hand, the content validity of the unstructured interview guidelines was determined using experts' agreement through scoring the items according to a graded scale. It was calculated with Aiken's formula to obtain the item validity index V of the high category, i.e., the value of V was found in the score range 0.6-0.8.

Procedure

The technical analysis of this qualitative research data consisted of processing and preparing data for analysis, condensation, and coding the data from the answer sheet, describing the types of errors and correction strategies, and the conclusion.

- *Data Analysis*

The quantitative data was found from the students' written responses to the test questions about angle and triangle definition (in Appendix) with the scores interval 0-100. Two researchers, separately and independently, identified the students who got low scores and had difficulties answering the test items. Each researcher made a list of the error (misconception) types conducted by students and computed the frequency number of each error type found. Based on these error types and their frequency number, the obtained data of both researchers were compared and re-examined. At the end of this evaluation, the researchers arrived at a mutual agreement that it was identified and determined five types of general errors of the students from high common errors level to low errors.

- *Identifying common errors and diagnosing the roots and the causes of misunderstanding*

Referring to the students' test answer errors included in these five error types, the researchers interviewed each student to discover his/her existing concept errors and misconceptions dealt with his/her formal definitions composed. Finding common errors and the roots of misunderstanding was classified and coded from high common errors level (E_1) to low errors (E_5), as shown in Table 1. Discovering the roots of their misconception, ensuring students mistake from the test answers (whether it was an error or misunderstanding), and improving awareness of students from the thinking, we interviewed them 1-1 through a personal approach related to their incorrect answers. The interview content was related to their experiences about the students' geometry pre-knowledge, ways of thinking, oral describing the definition of angle and triangle, and admitting his/her errors. The results of these activities were presented in Table 2. The understanding of the students and inadequate understanding in the category of minor misconceptions are respectively treated by defragmenting and connecting the incorrect geometry concepts with the right ideas. Other, it was treated by using reconstruction actions as shown in Table 3.

- *Misconceptions' Correction and Concepts Exchange Decision*

The treatment for the group of students with the minor error was carried out individually following the metacognitive regulatory mechanism in Figure 1. In starting tasks of misconceptions correction, the students should find out all primitive terms and some supporting concepts used to define the angle and triangle (point, line, ray, segment, position, and direction). Using these elementary geometry objects, they should try to draw and present any angle and triangle shapes. Hereafter, they explained each construction process and wrote his/her result work in the formal definitions using their language. In this case, the intervention and instruction of the educator emphasized improving and revising the following knowledge aspects. (1) Recognizing the pre-knowledge for defining the geometry concepts; (2) Demonstrating the concepts' construction process with pictures; (3) Presenting and writing this demonstrated concept idea; and (4) Composing the formal definitions. The corrective function of the treatments is to fill the gap (hole) of students' conceptual understanding or revise and reconstruct the concepts' wrong parts.

On the other hand, the treatment for students from the acute error group remediated the same as those in the minor misconception group. Still, we divided them into some groups (3 students per group). The group function was to discuss and evaluate the alternative definitions that resulted from the group members. Hereafter, each group should finally produce some geometry alternatives definitions that were most suitable with the formal concept discussed. During this step, the educator used some counterexamples and cognitive conflict strategies to make students dissatisfied with their ideas. These ways were also applied to straighten students' thinking, guide, and help them to exchange the students' incorrect concepts with correct concepts. The scheme and results of these treatments are introduced in Table 4.

The educator evaluated the students' difficulties correction results of both minor and major misconception levels. If it was considered that both groups' works were correct, clear, and coherent with the formal concepts, then they could pursue a new idea. If not, they should repeat in-deep.

Results

Problem 1: Identifying Common Errors and Diagnosing Roots and Causes

Based on the analysis results of student answers to ~~of~~ the test items in the Appendix, it was founded that thirty-seven students made errors in defining angle, sixty-one students in explaining triangle. In this case, this research identified five types of students' common errors. First, students assumed that an angle was a point (E_1). Second, they thought that an angle was the area part of a plane between two legs of this angle (E_2). Third, students argued

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that an angle was a figure represented by two line segments combined at one endpoint of both line segments (E_3). Fourth, they stated that a triangle was a part of a plane piece that forms the triangle (E_4), and, fifth error, they concluded that any three line segments defined a triangle (E_5). On the other side, in answering test item 1, there were 14 students (17.5%) of the error E_1, 11 students (13.8%) of the error E_2, and 12 students (15%) of the error E_3. For the test items 2 and 3, respectively, there were 28 students (33.8%) of the error E_4 and 33 students (41.3%) of the error E_5. Thus, the total errors of the test items 1, 2, and 3 were successively 37 students (46.3%), 28 students (33.8%), and 33 students (41.3%). The misconceptions frequencies (f) of these thirty-seven students in solving geometry test items 1, 2, and 3 see in Table 1.

Table 1. Frequencies of Students' Misconception in Understanding Angle and Triangle

Problems	**Error 1 (E_1)**		**Error 2 (E_2)**		**Error 3 (E_3)**		**Error 4 (E_4)**		**Error 5 (E_5)**		**Total**	
	f	%	f	%	f	%	f	%	f	%	f	%
Test item 1	**14**	17.5	**11**	13.8	**12**	15	-	-	-	-	**37**	46.3
Test item 2	-	-	-	-	-	-	**28**	33.8	-	-	**28**	33.8
Test item 3	-	-	-	-	-	-	-	-	**33**	41.3	**33**	41.3

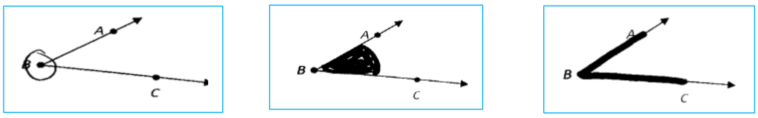
The errors' roots and causes of students' misconceptions have resulted from the student works analysis and in-depth interviews. The interview content was related to their experiences about geometry pre-knowledge, ways of thinking, oral describing and writing the definitions of angle and triangle, and admitting errors. From the interview results, we also evaluated the missing and incomplete knowledge structures, the connection among the learned material of the students, and the logical consequences as follows.

Misconception 1: Over-specializing that an angle $\angle ABC$ is the point *B*.
Students who underwent this misconception could memorize that an angle definition $\angle ABC$ was the union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersected at the common endpoint *B*, but they marked point *B* as an angle (Figure 2a). Regarding results of interviews with the students of the misconceptions E_1, they did not generally understand some undefined terms and geometry's elementary objects used for defining rays, line segments, and drawing an angle. They could not explain the relationship between these concepts to construct the angle either. Moreover, they ignored the starting point position and the direction for a line ray and habited the writing angle symbol with only one capital letter. Due to the students define the angle maybe with rote, consequently, they argued that this angle is the point *B*.

Misconception 2: Over-generalizing that an angle is the area part of a plane bounded by two legs of this angle.
This misunderstanding appeared in the students' answers to Test Item 1 about the problem of angle. Although the students had already known an angle constructed by two rays intersect at the endpoint, they stated an angle figure as the area bounded by their angles legs. For example, Figure 2b shows the work of a student who has this second misconception type. The in-deep interviews could inform the students' misconceptions from two causes. They did not understand that two rays met at the starting point would consistently result in the rays pieces connection (not a cut of plane), and they lacked the prior knowledge to differentiate between an angle and its measure. As a result, they had the misconception that an angle was the area part of a plane bounded by two legs of this angle.

Misconception 3: An angle is a figure formed by two-line segments that meet at one endpoint of the segments.

In the third misconception, the students recognized that the angle $\angle ABC$ was a union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ coincided at its starting point B, but they stated that an angle $\angle ABC$ was two line segments $\overline{BA}$ and $\overline{BC}$ that met at point B. For example, Figure 2c shows the works result of students that the line segments $\overline{BA}$ and $\overline{BC}$ as an angle. They argued an angle $\angle ABC$ was a set of points of line segments $\overline{BA}$ and $\overline{BC}$. Appertaining to the results of in-depth interviews, the students who committed the misconceptions had not consistently differentiated between segments and rays to define an angle. They over-specialized this angle represented with three points and three capital letters. Because of this, the students said an angle was a figure formed by two-line segments that met at one endpoint of the segments.



(a) (b) (c)

Figure 2. Student work results in the completion of the test items 1

Misconception 4: A triangle is a part of a plane piece that forms the triangle.
Students considered a triangle as the interior of the triangle or the area bounded by the triangle. This misconception appeared in the students' works of test item 2 relating to the triangle concept. In this test item 2, the students who had a misunderstanding could define a triangle as a polygon of three sides but pointed out the graph that the triangle was an area bounded by the sides of the triangle (interior of the triangle). For example, Figure 3a shows the works of students experiencing the fourth misconception. Referring to the results of interviews, the students' error E_4 did not know that the merging three-line segments at its endpoints for constructing triangle would produce three line-segments connection picture. They also used their primary school experiences in which a triangle was made from cutting paper through three noncollinear points. In consequence, these students declared a triangle was a part of a plane piece that forms the triangle.

Misconception 5: Any three-line segments define a triangle.
Students concluded that any three-line segments can form a triangle. This misconception happened from test item 3 about three-line segments as data for triangle sides. Students understood that triangle sides had three-line segments. Relating to the solution of test item 3, Figure 3b, the students explained that any three-line segments could form a triangle, i.e., a right triangle or other triangles. Because of these triangle images and without examining the measure of these three-line ~~three line~~ segments data, they made wrong conclusions.

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(a) (b)

Figure 3. Students Work Results in the Completion of the Test Items 2 and 3

Appertaining to these students results of interview, the errors E_1 to E_5 found ten information about the roots of students' misconceptions. The list of these roots and causes of students' misunderstanding presents in Table 2.

Table 2. Common Errors, the Roots and the Causes of Students' Misunderstanding

Code	Common errors	The Roots and the Causes of Misunderstanding (Code)
E_1	An angle $\angle ABC$ is a point B.	1. The students do not understand some undefined terms and geometry's elementary objects used for defining rays, line segments, and drawing an angle (E_{11}). 2. They do not know the relation between these definitions to construct an angle (E_{12}). 3. They define the angle with the rote and habit the writing angle symbol with only one capital letter (E_{13}).
E_2	An angle is an area part of a plane bounded by two legs of the angle.	1. The students do not understand that if two-line rays meet at their starting point will consistently result in the rays' pieces connection (E_{21}). 2. They lack the prior knowledge to differentiate between an angle and its measure or the area between their angle legs (E_{22}).
E_3	An angle is a figure formed by two line segments that meet at one endpoint of the segments.	1. The students cannot consistently differentiate between segment and ray to define an angle (E_{31}). 2. They over-specialize an angle represented with three points and three capital letters (E_{32}).
E_4	A triangle is a part of a plane piece that forms the triangle.	1. The students do not know the merging three-line segments at their endpoints for constructing a triangle will produce a line-segments connection picture (E_{41}). 2. They have a misconception from primary school experiences in which a triangle makes from cutting paper through three noncollinear points (E_{42}).
E_5	Any three line segments define a triangle.	1. The students state the conclusion for defining a triangle using some triangle images without counting and comparing the length of three-line segments (E_{51}).

Problem 2: Misconceptions' Correction and Concepts Exchange Decision

In general, we found the sources of these misconceptions of students were the lack of prior knowledge or missing knowledge (MK) of geometry concepts, the existence of knowledge gap or incomplete knowledge (IK), interpretation deviation (ID) of concept images, feeble-logical thinking (FT), and low connection (LC) of students' knowledge. These causes characterize the misconceptions in the following three types. In the case of causes of MK and IK, we call inadequate understanding with minor errors (10 students). For the causes of ID and FT, we state inadequate understanding with major mistakes (18 students), another (LC), it calls almost understanding with minor errors (9 students). Resolving these misconceptions' causes, we introduce the approach to reconstruct thinking fragmentations of MK and IK, rearrange knowledge structures and logical thinking of ID and FT, and link knowledge of students LC. For this solutions approach, we believe, respectively, with the terms: defragmentation, reconstruction, and connection solutions as shown in Table 3.

Table 3: Treatment Approaches of Students' Misunderstanding

Concepts Mastery Achievement	**Number and kinds of Students Misconception and Treatment Types**	
	Minor	**Major**
Inadequate understanding	10 Students of MK and IK *Defragmentation*	18 Students of ID and FT *Reconstruction*
Almost understanding	9 Students of LC *Connection*	-

This section reports the errors correction of the students. It cures the students' misunderstanding roots in Table 2 for their error cases in Table 1. The stages were as follows (Table 4). Implementing the metacognitive regulation scheme presented in Figure 1 is labeled M. The instructor directed the students to learn primitive concepts of the angle or triangle (activity M1) and design figures of the angle or triangle (activity M2). Evaluating these students' activities was focused on drawing an angle or a triangle idea connected with primitive concepts. The stage of activities M1 and M2 is called the drawing concept and labeled D. Then, the students presented pictures and wrote geometry ideas based on prior knowledge (activity M3) and composed formal definitions (activity M4). The instructor helped the students to recognize the geometry characteristics and connections of the angle and triangle elements, the logical thinking for constructing an angle or triangle and write the definitions of angle and triangle through the pictures. The stage of activities M3 and M4 is called the writing task and labeled W. During the learning activities D and W, the instructor gave interventions and instructions to develop the students' knowledge structure and induce cognitive conflicts. These cognitive conflicts were designed to resolve the causes of students' misunderstanding MK, IK, ID, FT, and LC. The intervention actions of the cognitive conflict, i.e., respectively, marked by the code I_{MK}, I_{IK}, I_{ID}, I_{FT}, and I_{LC} presented in columns 4-7 in Table 4. Students with minor misconceptions were individually treated, in contrast, to the major misconceptions, remedied in group of five students. Assessing students' achievement with scores interval 0–100 and in-depth interview, the treatments found the average result scores shown in column nine of Table 4. These remedial activities M, D, W, and I are called metacognition/drawing/writing/intervention (MDWI) strategy.

Table 4: Remedial Errors

Errors Roots	**Students Number**	**Treatment Types**	**Metacognitive Regulation (M)**				**Correct Students Number**	**Average Result Scores (0 – 100)**
			Drawing Concept (D)		**Writing Task (W)**			
			M1	M2	M3	M4		
E_{11}	**7**	Defragmentation, Reconstruction, & connection.	I_{MK}	I_{IK}	I_{ID}	I_{FT}-I_{LC}	**5**	**71.43**
E_{12}	**4**	Reconstruction.	-	-	I_{ID}-I_{FT}	I_{FT}	3	**75.00**
E_{13}	**3**	Reconstruction & connection.	-	-	I_{ID}	I_{FT}-I_{LC}	2	**66.67**
E_{21}	**6**	Reconstruction	-	-	I_{ID}	-	4	**66.67**

E_{22}	**5**	Defragmentation & reconstruction.	I_{MK}-I_{IK}	I_{ID}	I_{ID}	-	4	**80.00**
E_{31}	**7**	Defragmentation & reconstruction.	I_{MK}	I_{ID}	I_{ID}	-	5	**71.43**
E_{32}	**5**	Reconstruction.	-	-	I_{ID}	I_{FT}	4	**80.00**
E_{41}	**17**	Reconstruction & connection.	-	-	I_{ID}-I_{LC}	-	13	**76.47**
E_{42}	**11**	Reconstruction.	-	-	I_{ID}	-	8	**72.73**
E_{51}	**33**	Reconstruction & connection.	-	-	I_{ID}	I_{LC}	24	**72.73**
Total	**98**	**Number of Interventions**	**4**	**3**	**12**	**7**	**72**	**73.31**

Some cognitive conflicts examples associated with the treatments of students' misunderstanding MK, IK, ID, FT, LC in Table 2 and often used by the instructor for intervening students in this research were as follows.

1. An angle $\angle ABC$ is just a point B that is undefined geometry object; versus a set of points consisting of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersects at the endpoint *B*.
2. An angle $\angle ABC$ is a measure of arc degree or an area between two angle legs $\overrightarrow{BA}$ and $\overrightarrow{BC}$; contra to the joint of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ coincides at the point *B*.
3. An angle $\angle ABC$ is a joint of two-line segments $\overline{BA}$ and $\overline{BC}$ meet at the endpoint *B*; against the union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersects at the endpoint *B*.
4. An angle $\angle ABC$ is just three points *A*, *B*, and *C*; versus a union of two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ meets at the point *B*.
5. The joining of three-line segments $\overline{BA}$, $\overline{BC}$, and $\overline{AC}$ at their endpoints *A*, *B*, and *C* will form a cut of a plane *ABC* called a triangle *ABC*; contra to they produce a picture of three-line segments called a triangle *ABC*.
6. Every three-line segments $\overline{BA}$, $\overline{BC}$ and $\overline{AC}$ can form a triangle *ABC*; contra to the line segments of the measures $\overline{BA}$ = 3 cm, $\overline{BC}$ = 5 cm, and $\overline{AC}$ = 15 cm will not construct a triangle *ABC*.

Discussion

Based on Table 1 and Table 2, it informs that only 43 students (53.7%) understood the concept correctly, 37 students (46.3%) could not define the angle, and 33 of the 61 students (76.3%) failed to understand the triangle idea. Moreover, we found no more than 54% of the students could correctly define the angle and triangle terms. Main impediments and feebleness of the students were that they did not know the geometry primitive terms, the function, and the role of the ray and line segment in defining an angle or triangle. In this case, they might learn the geometry concepts and definitions partially. They also underwent the visual deviation between a point and an angle, an angle measure and an angle, and between a triangle and a plane cut triangle. These results were relevant to the studies of Ozkan et al. (2018), Al-Khateeb (2016), and Özerem (2012). They informed that one of the misconceptions causes was the lack of prior knowledge and insufficient students' knowledge of the geometry concepts. These finding results were also in line with the studies of Poon and Leun (2016), Biber et al. (2013), Cunningham and Roberts (2010), and Gal and Linchevski (2010) who found that students faced difficulties in selecting the characteristics of figural elements relevant to the concepts. Due to the fact that students focused only on the physical shapes and the geometry images, they had difficulties in identifying the essential geometry properties of represented figures and fundamental logical reasoning abilities.

The mathematical material is, generally, interconnected. The students who do not understand the geometry relations between the points, lines, rays, and line segments will find trouble stating an angle or a triangle. Thus, to correctly define an angle and triangle, they must mainly understand the idea, the function, and the role of these elementary geometry objects.

Handling the students' misconceptions as presented in Table 4 shows that it is necessary for at least 26 interventions provided by the instructor. In general, students' misconceptions are the interpretation deviation of the image concept to define angle or triangle. The source of this main problem found that the students fail to recognize the geometry objects for drawing the angle or triangle and flunk to clarify, logically of how to construct this angle or triangle by using these objects. As the results, they misperceived that an angle was a point, three points, or a combination of line segments. Moreover, they made a wrong generalization (over-specializing and over-generalizing).

Correcting student misconceptions with the metacognition regulation scheme (M), the students' actions to draw all figural concepts (D), the writing ideas (W), and the intervention of the lecturer (I) shown in Table 4 improved the students' achievement score of 73 (very good), and the students of the correct answers of 72/98 = 73 %. It was found that this MDWI strategy could help the students to change their misconceptions in defining angle and triangle from misconception concepts to be correct concepts. It could also resolve the student errors in generalization, concept images, and connection incompetency of geometry features that were discussed by Gutiérrez and Jaime (1999), Özerem (2012), Poon and Leun (2016), Ozkan et al. (2018), and Şahin et al. (2020).

This introduced MDWI approach encourages students to revise their existing preconception errors and accommodate the new idea. Hopefully, it can give students opportunities to reflect and negotiate true mathematical meaning with their misconceptions and help them become stronger students. They will be more ready to replace their incorrect geometry concepts with correct concepts and decisions. Applying this MDWI strategy helps to defragments and reconstructs or substitutes the student's misconception ideas and an understanding gap with natural ways and considering of following aspects. It involves the motivation and beliefs, prior knowledge, and cognitive engagement related to students; content knowledge, interests, and teaching strategies related to the educator; the role of peer learning and students-educator relation associated with the social context. The students' eliminated errors correction includes misconceptions of generalization, concept images, geometry features, and properties (Figure 4).

These students' corrections result in averages come across that their works have been corrected and coherent with the formal definitions. Each student's score was more than 60. Thus, they could pursue a new learning topic.

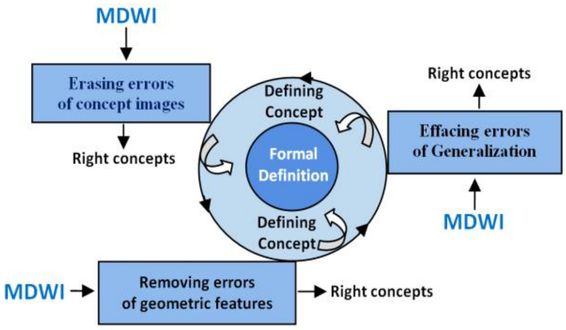

Figure 4. Errors Elimination

Conclusion

The common errors occurred in defining angle and triangle of mathematics education students, i.e., the assumption and generalization that angle was a point, an area part of a plane between two legs of the angle, and a union of two-line segments that meet at one endpoint of the segments. Then, they made the misconceptions that a triangle was a part of the plane piece forms the triangle, and any three-line segments define a triangle. The roots of these errors, generally, are of basic ideas. They did not know the undefined terms, the function, and the role of the ray and the line segment for defining an angle or triangle. The students commonly failed in concept images interpretation, logical thinking, and knowledge connection needed to draw, construct, and write the definitions of angle and triangle.

The MDWI approach could guide the students to learn primitive concepts, draw the geometry idea connected with primitive ideas, present and write these pictures into formal definitions. In other side, the instructor could give interventions and instructions to develop their knowledge structure, induce cognitive conflicts, and resolve the causes of their misunderstanding relating to the generalization errors, interpretation deviation of concept images, and connection incompetency of geometry features. During the remedial process, these treatments effects might empower and motivate the students to decline and reduce their errors in recognizing the geometry objects and generalization, interpretation concept images, and connection of geometry features.

This study shares new findings related to identifying the errors, the roots, and the causes of students' misunderstanding in defining geometry concepts, particularly the angle and triangle. This study contributes knowledge for lecturers and researchers about how correcting students' misconception are conducted by implementing metacognitive skills, drawing geometry concepts, writing geometry ideas connected with the prior knowledge, and interventing students during the learning process activities. The research emphasizes that the use of MDWI approach can help correct students' misconceptions in defining these geometry ideas.

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Recommendations

Considering error correction is rare in defining geometry objects, this MDWI treatment provides a guide for curing the students of the geometry concepts misconception step by step based on the metacognitive regulation. In line with the university students' errors amount of these generalizations and concept images, the treatment activities M2 and M3 are more effective for helping them avoid the misunderstanding of interpretation deviation and logical thinking of concept images and geometry features.

In general, geometry learning materials in secondary school emphasize mastery of geometry concepts related to the definition of geometric objects, measurements, and calculation of the area and volume of these objects. On the other hand, the assessment reports in mathematics learning, particularly geometry, showed low completeness. Based on the study results, it recommends that further research should be conducted on junior and high school students to identify the cause and the roots of the difficulties in mastering geometry ideas. Besides, employing the MDWI treatment helps the students remedy their misunderstandings caused by a lack of prior knowledge, incomplete knowledge, interpretation deviation of concept images, feeble-logical thinking, or a low connection of students' prior knowledge.

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Limitations

This research was conducted on students elected from a private university in NTB Province, Indonesia. Thus, the generalization of the results in this research has limitations. Another limitation of the study was that it only focused on defining the terms of geometry concepts.

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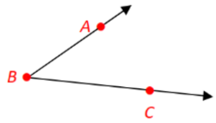
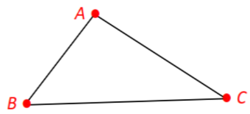
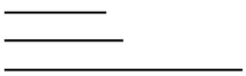
Appendix

Instruments for Identifying Angle and Triangle Misconceptions and Types of Problems

Problem	**Problems Types**
1. Consider the points *A*, *B*, and *C* in Figure 1. Give a mark using a colored pen, which is a part of Figure 1 called an angle? Give your reason in detail!  Figure 1. Angle	Angle Problem
2. Let three points *A*, *B*, and *C* in Figure 2. Give a mark using a colored pen, which is a part of Figure 2 called a triangle? Give your reason in detail!  Figure 2. Triangle	Triangle Problem
3. Given any three line segments in Figure 3. Can these line segments form a triangle and explain why!  Figure 3. Three line Segments	Line Segments Problem on Triangle